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Contributed Paper

Alternative Tests for the Poisson Distribution

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ABSTRACT

Poisson distribution is well used as a standard model for analyzing count data. Tests based on the index of dispersion (ID) concerning the variance and the mean are not able to discriminate between the Poisson distribution and some others. An alternative method for the test that data comes from Poisson distribution are proposed based on skewness properties, which is the same as coefficient of variation properties. Monte Carlo studies show that the proposed tests are powerful competitor to the ID tests when alternative distribution is not Poisson but its variance is close to the mean.

Keywords: coefficient of variation, goodness of fit test, index of dispersion, skewness

1. INTRODUCTION

In many applications, the variable of interest is given in the form of an event count or a nonnegative integer value which refers to the number of occurrences of a particular phenomenon over a fixed set of time, distance, area or space. Some examples of such data are the number of road accident victims per week, number of cases with a specific disease in epidemiology, etc. Poisson distribution is a standard and good model for analyzing count data and it seems to be the most common and frequently used as well. Equality of mean and variance is an important characteristic of Poisson family of distributions. In practice, a sample with equal dispersion is rare, i.e., over-dispersion (sample variance is greater than sample mean), and under-dispersion (sample variance is less than sample mean) usually occur even when a random sample is drawn from a Poisson distribution. On the other hand, equal dispersion is held by many other distributions such as beta-binomial distribution with parameters r=5, a=1 and b=2/3 and discrete uniform distribution with support in 0, 1, 2, ... , r , e.g. r=4.

It is very interesting to be able to test whether a sample is drawn from a Poisson distribution. Suppose $X_1, X_2, \ldots, X_n$ is a random sample of size n from a discrete distribution. A problem of testing for Poissonity is given by

$$H_0: X \text{ is distributed as Poisson } (\lambda),\ \lambda \in (0,\infty),$$

against

$$H_1: X \text{ is not distributed as Poisson.}$$

Several methods have been proposed to test for goodness of fit of a Poisson distribution which can be divided into 4 groups. The first group consists of tests developed from some characteristics on different orders of moment of the Poisson family. The very first test in this group is based on the index of dispersion (ID, variance-to-mean ratio) proposed by Fisher *et al.* [1] and is defined by $D = \frac{(n-1)M_2}{\bar{X}}$, which is distributed approximately, under the null hypothesis, as chi-squared with n – 1 degrees of freedom. Note that $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$ and $M_2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2$. Gart and Pettigrew [2] have proposed 3 test statistics based on the conditional moment of the cumulants of Poisson distribution as follow:

$$Z_2 = \frac{\frac{nm_2}{(n-1)} - \bar{X}}{\sqrt{\frac{2\bar{X}(n\bar{X}-1)}{n(n-1)}}},$$

$$Z_3 = \frac{\frac{n^2 m_3}{(n-1)(n-2)} - \bar{X}}{\sqrt{\frac{6\bar{X}(n\bar{X}-1)}{n(n-1)}\left[3 + \frac{(n\bar{X}-2)}{(n-2)}\right]}} \text{ and}$$

$$Z_4 = \frac{\frac{n^2\left[(n+1)m_4 - 3(n-1)m_2^2\right]}{(n-1)(n-2)(n-3)} - \bar{X}}{\sqrt{\frac{2\bar{X}(n\bar{X}-1)}{n(n-1)}\left[49 + \frac{108(n\bar{X}-2)}{(n-2)} + \frac{12(n+1)(n\bar{X}-2)(n\bar{X}-3)}{n(n-2)(n-3)}\right]}}$$

where $m_r = \frac{1}{n}\sum_{i=1}^{n}(X_i - \bar{X})^r$.

These 3 test statistics are asymptotically normally distributed as $n \to \infty$. Böhning's test [3] is one of the well – known tests which is impossible to ignore. The test statistic is defined on difference between variance and mean of a sample as $BN = \sqrt{\frac{n-1}{2}}\left(\frac{M_2}{\bar{X}} - 1\right)$ and is also asymptotically normal distributed as $n \to \infty$. Last but not the least, to be mentioned is the test based on equality of squared skewness and kurtosis proposed by Gupta *et al.* [4], $G = \sqrt{\frac{n}{1+24\bar{X}+6\bar{X}^2}}\left(\frac{m_2(m_4 - 3m_2^2) - m_3^2}{2\bar{X}^2}\right)$. Asymptotic distribution of G was shown to be a standard normal. The second group of tests for Poissonity consists of tests based on an empirical distribution function as discussed by several authors and reviewed by Karlis and Xekalaki [5]. The third group of tests is based on the probability generating function (PGF) such as the test of Kocherlakota and Kocherlakota [6] which relies on the difference between Poisson PGF and its empirical PGF and the value of t, and many other developed tests as has been reviewed by Gürtler and Henze [7], and the last group consists of other existing tests for Poissonity such as Brown and Zhao [8] have proposed a test based on Anscombe's variance stabilizing transformation.

Simulation techniques have been used to compare between various tests for Poissonity in the literatures. The Böhning's test seems to be preferred [4] if a Poisson hypothesis is tested against either over-dispersed or under-dispersed alternative while it fails against equally dispersed alternatives. In addition, Fisher's ID test obtained high power in almost all cases except the case where ID was closed to 1 [7]. The above findings exhibit that these two well – known tests, Fisher's ID and Böhning's test, perform very well when testing against a family of distributions characterized by non – equal dispersion but not vice versa. Hence, two alternative tests for Poissonity based on the properties of sample skewness and the coefficient of variation are proposed in this paper so that Poisson distribution can be discriminated from another distribution having equal or almost equal dispersion.

2. GOODNESS OF FIT TEST FOR POISSONITY

2.1. Useful Properties of Poissonity

Besides equal dispersion, another interesting characteristic of the Poisson family is that the skewness and coefficient of variation (CV) are equal to the reciprocal square root of its mean. Skewness of a distribution of a random variable *X*, in general, is measured by coefficient of skewness which is denoted by γ_1 and is defined as follow.

$$\gamma_1 = \frac{\mu_3}{\sqrt{\mu_2^3}}. \quad (2.1)$$

where μ_i represents the i[th] central moment of X. If the coefficient of skewness is positive, the distribution is skewed to the right that is the distribution has long right tail. If it is negative, then the distribution is skewed to the left. For the family of Poisson distributions with parameter $\lambda > 0$, the 2[nd] central moment or the variance of X is $\mu_2 = \lambda$ and the coefficient of skewness is equal to $\lambda^{-1/2} > 0$. Then, from equation (2.1), $\mu_3\mu_2^{-3/2} = \lambda^{-1/2} = \mu_2^{-1/2}$ and thus

$$\mu_3 - \mu_2 = 0. \tag{2.2}$$

A CV is a measure of variability and is known to be independent of scale or, in other words, it isn't affected by the units of measurement. In particular, the CV of the Poisson family with parameter λ, is

$$CV = \frac{\sqrt{\mu_2}}{\mu} = \lambda^{-1/2}. \tag{2.3}$$

Here again $\mu_2 = \lambda$ for the Poisson family equation (2.3) can be rewritten as $\mu_2 - \mu = 0$, and hence, subtracting the latest equation by equation (2.2) attains

$$2\mu_2 - (\mu_3 + \mu) = 0. \tag{2.4}$$

Equation (2.2) and (2.4) are the first two conclusion of properties of Poisson family of distributions. It is possible to verify that the unbiased estimators of $\mu_3 - \mu_2$ and $2\mu_2 - (\mu_3 + \mu)$ are, respectively, $S_1 = M_3 - M_2$ and $S_2 = 2M_2 - (M_3 + \bar{X})$, where

$\bar{X} = (1/n)\sum_{i=1}^{n} X_i$, $M_2 = (1/(n-1))\sum_{i=1}^{n}(X_i - \bar{X})^2$ and

$M_3 = (n/(n-1)(n-2))\sum_{i=1}^{n}(X_i - \bar{X})^3$. Consequently,

$$E[M_2] = E[M_3] = \lambda, E\left[(M_2 - \mu_2)^2\right] = \frac{\lambda}{n}\left[1 + \frac{2n\lambda}{(n-1)}\right],$$

$$E\left[(M_3 - \mu_3)^2\right] = \frac{1}{n}\left[\lambda + \frac{18n\lambda^2}{n-1} + \frac{6n^2\lambda^3}{(n-1)(n-2)}\right],$$

$$E\left[(\bar{X} - \mu)(M_2 - \mu_2)\right] = E\left[(\bar{X} - \mu)(M_3 - \mu_3)\right] = \frac{\lambda}{n}$$

and $E\left[(M_3 - \mu_3)(M_2 - \mu_2)\right] = \frac{1}{n}\left[\lambda + \frac{6n\lambda^2}{n-1}\right]$.

Accordingly, the follows thus simply verified.

Theorem 1. *Let $X_1, X_2, \ldots, X_n$ be independent and identically distributed random variables from a Poisson distribution with mean λ. Then*

$$(i)\ E[S_1] = 0 \text{ and } V[S\ ] = \frac{2\lambda}{n-1}\left[4 + \frac{3n\lambda}{n-2}\right]. \tag{2.5}$$

$$(ii)\ E[S_2] = 0 \text{ and } V[S_2] = \frac{2\lambda^2}{n-1}\left[1 + \frac{3n\lambda}{n-2}\right]. \tag{2.6}$$

$$(iii)\ T_1 = \frac{2\bar{X}(n\bar{X}-1)}{n(n-1)}\left[4 + \frac{3n\bar{X}-6}{n-2}\right] \text{ and}$$

$$T_2 = \frac{2\bar{X}(n\bar{X}-1)}{n(n-1)}\left[1 + \frac{3n\bar{X}-6}{n-2}\right],$$ *are consistent estimators, respectively of* $V[S_1]$ *and* $V[S_2]$.

2.2 Proposed Tests for Poissonity and Their Asymptotic Distributions

The proposed tests for Poissonity are based on the statistics S_1 and S_2 and their asymptotic distributions are verified in Theorem 2.

Theorem 2. *Let $X_1, X_2, \ldots, X_n$ be a random sample of size n selected from a Poisson distribution with mean λ and let*

$$SK = \frac{M_3 - M_2}{\sqrt{\frac{2\bar{X}(n\bar{X}-1)}{n(n-1)}\left[4 + \frac{3n\bar{X}-6}{n-2}\right]}} \tag{2.7}$$

and

$$CSK = \frac{2M_2 - (M_3 + \bar{X})}{\sqrt{\frac{2\bar{X}(n\bar{X}-1)}{n(n-1)}\left[1 + \frac{3n\bar{X}-6}{n-2}\right]}}. \tag{2.8}$$

Then $SK \xrightarrow{D} N(0,1)$ *and* $CSK \xrightarrow{D} N(0,1)$ *as* $n \to \infty$.

Proof. Since $S_1 = M_3 - M_2$ and $S_2 = 2M_2 - (M_3 + \bar{X})$has continuous functions in the neighborhood of (μ_2,μ_3) and (μ,μ_2,μ_3), respectively. By the Theorem 8.16 of Lehmann and Casella [9], we have

$$\left[M_3 - M_2\right] \xrightarrow{D} N(0,\sigma_1^2) \text{ and}$$

$$2M_2 - (M_3 + \bar{X}) \xrightarrow{D} N(0,\sigma_2^2),$$

where $\sigma_1^2 = \frac{2\lambda^2}{(n-1)}\left[4 + \frac{3n\lambda}{(n-2)}\right]$ and

$$\sigma_2^2 = \frac{2\lambda^2}{(n-1)}\left[1 + \frac{3n\lambda}{(n-2)}\right].$$

Hence, by Theorem 1 and conclusions of the theorem follow from Slutsky's theorem for fixed λ as $n \to \infty$.

According to Theorem 2, the two appropriate test statistics for Poisson distribution testing, SK and CSK, as defined in equation (2.7) and (2.8), respectively, are then proposed. If a distribution from which a sample is drawn is a Poisson distribution, then SK (and CSK) should take on a small value (closed to zero), and vice versa. Therefore, the null hypothesis of Poissnity would be rejected at level α when $|SK| > Z_{\alpha/2}$ and $|CSK| > Z_{\alpha/2}$, where $Z_{\alpha/2}$ is the upper $\alpha/2$ quantile of the standard normal distribution.

3. Empirical Results

This section presents some selected empirical results for testing Poissonity. The two proposed tests, SK and CSK, respectively, will be compared to five other tests, i.e., Fisher's test (D), Böhning's test (BN), the two tests of Gart and Pettigrew (Z_3 and Z_4) and Gupta's test (G). The power of these tests are evaluated by a Monte Carlo simulation using 10,000 replications for a sample of moderate size (n = 50 and 100) and the significant level to be considered is $\alpha = 0.05$, provided by the statistical package R [10]. Many alternative discrete distributions with over-, under- or equal-dispersion are taken into account where the involved parameters are chosen so that the means of all distributions above are closest to the mean of the Poisson distribution under the null hypothesis, $\lambda \in (0,5]$ [11]. Some properties of these distributions are exhibited in Table 1.

To investigate the sampling distribution of the two proposed tests, we used the quantile plot (Q-Q plot) in an analogous way as Baksh *et al.* [12]. Figure 1, as shown, for the sampling distribution investigating of the two proposed tests (SK and CSK, respectively) against the BN test, seeing that the two proposed tests are also converge to standard normal distribution as n tends to infinity.

In Table 2, for the null hypothesis of Poissonity is true, the test statistics D, BN, Z_3, SK and CSK are quite accurate in the sense that the empirical Type I error rate in all cases is 0.046 – 0.052 which coincides to the desired significant level ($\alpha = 0.05$). While the empirical Type I error rates of the tests which rely on the high central moments (Z_4 and G) never reach the nominal level. The estimated powers of all tests at the level 0.05 are always high power as the sample size increases.

Under-dispersed alternative with binomial distribution, the empirical power of the D test is greater than those of the other investigated tests, though it seems to be not much high when n = 50. While the rest of the tests (i.e., Z_3, Z_4, G, SK and CSK), are found to be very poor.

Over-dispersed alternatives include the negative binomial (NB) and zero-inflated Poisson (ZIP). All two tests related to the index of dispersion (ID) namely D and BN seem to be outstanding in terms of empirical power for NB and ZIP distributions. The two proposed tests, SK and CSK, attain a lower power than those two tests based on ID for NB distributions and the power of CSK increases as n increases for ZIP distribution.

Mixed-dispersion alternatives with discrete uniform (DU), beta-binomial (BB)

Table 1. Some characteristics of selected discrete distributions.

Type of discrete Distribution	Mean	Variance	ID	Skewness
1. Under-dispersed				
Binomial: Bin$\left(r,\frac{\lambda}{r}\right)$	λ	$\lambda-\frac{\lambda^2}{r}$	$1-\frac{\lambda}{r}$	$\frac{1-\frac{2\lambda}{r}}{\sqrt{\lambda\left(1-\frac{\lambda}{r}\right)}}$
2. Over-dispersed				
Negative binomial: NB$\left(r,\frac{r}{r+\lambda}\right)$	λ	$\lambda+\frac{\lambda^2}{r}$	$1+\frac{\lambda}{r}$	$\frac{r+2\lambda}{\sqrt{r\lambda(r+\lambda)}}$
Zero-inflated Poisson: ZIP$\left(\frac{\lambda}{1-p},p\right)$	λ	$\lambda\left(1+\frac{p\lambda}{1-p}\right)$	$1+\frac{p\lambda}{1-p}$	$\frac{(1-p)(1-p+3p\lambda)-(1-2p)p\lambda^2}{\sqrt{\lambda(1-p)(1-p+p\lambda)^3}}$
3. Equi- dispersed				
Poisson: Poi(λ	λ	λ	1	$\frac{1}{\sqrt{\lambda}}$
4. Mixed-dispersed				
Beta binomial: BB$\left(r,a,\frac{(r-\lambda)a}{\lambda}\right)$	λ	$\lambda\left(1-\frac{\lambda}{r}\right)\left(1+\frac{(r-1)\lambda}{ra+\lambda}\right)$	$\left(1-\frac{\lambda}{r}\right)\left(1+\frac{(r-1)\lambda}{ra+\lambda}\right)$	$\frac{(r-2\lambda)(a+2\lambda)}{(ra+2\lambda)\sqrt{\lambda\left(1-\frac{\lambda}{r}\right)\left(1+\frac{(r-1)\lambda}{ra+\lambda}\right)}}$
Discrete uniform: DU(2λ)	λ	$\frac{\lambda(\lambda+1)}{3}$	$\frac{\lambda+1}{3}$	
Zero-inflated Generalized Poisson: ZIGP$\left(\frac{(1-\theta)\lambda}{(1-p)},\theta,p\right)$	λ	$\frac{\lambda\left[1-p+(1-\theta)^2p\lambda\right]}{(1-p)(1-\theta)^2}$	$\frac{1-p+(1-\theta)^2p\lambda}{(1-p)(1-\theta)^2}$	$\frac{(1+2\theta)(1-p)^2+3(1-\theta)^2(1-p)p\lambda+(1-\theta)^4(2p-1)p\lambda^2}{(1-\theta)\sqrt{(1-p)\lambda\left(1-p+(1-\theta)^2p\lambda\right)^3}}$

and zero-inflated generalized Poisson (ZIGP) distributions. The CSK test appears to perform very well under all mixed alternatives when ID is close to or higher than 1. While the two tests related to ID (i.e., D and BN) have very low power. The Z_3 is high power for all cases of the BB distributions when n increases.

4. CONCLUSION AND DISCUSSION

In general, tests based on the index of dispersion (ID) concerned the first two moments of distribution, mean and variance, are outperformed against many alternatives when ID is far from 1 [4,8]. However, the tests based on the ID sometimes cannot be discriminate between Poisson and other distributions such as discrete uniform and beta-binomial distribution, etc. The findings in this paper agree with this conclusion. In this paper, two proposed alternative tests are proposed based on the properties of sample skewness, which is the same as the coefficient of variation, are called SK and CSK. Simulation results shown that the estimated values of size of the SK and CSK tests at 5% level are closed to the significant level in all cases and are similar to the two test related to ID (i.e., D and BN). The power of all two tests related to ID (D and BN) are powerful under several circumstances except when ID is close to 1. However, when the ID is close to 1, the SK and CSK tests are more powerful than

Table 2. Simulated values of power of various tests for Poissonity at significance $\alpha = 0.05$ against alternatives when n = 50 and 100 with 10,000 replications.

n	Distributions	mean	Variance	ID	Test Statistic						
					D	BN	Z_3	Z_4	G	SK	CSK
	Poi(*1*)	*1.00*	*1.00*	*1.00*	0.049	0.051	0.051	0.041	0.009	0.051	0.047
	Poi(*3*)	*3.00*	*3.00*	*1.00*	0.050	0.050	0.049	0.039	0.023	0.048	0.046
	Poi(*5*)	*5.00*	*5.00*	*1.00*	0.052	0.047	0.048	0.043	0.034	0.049	0.051
	Bin(*20, 0.15*)	*3.00*	*2.55*	*0.85*	0.102	0.062	0.009	0.011	0.002	0.010	0.013
	NB(*20, 0.87*)	*3.00*	*3.45*	*1.15*	0.123	0.151	0.134	0.091	0.073	0.122	0.106
	ZIP(*3.15, 0.048*)	*3.00*	*3.45*	*1.15*	0.123	0.151	0.063	0.047	0.094	0.056	0.081
	DU(*3*)	*1.50*	*1.25*	*0.83*	0.062	0.028	0.001	0.000	0.011	0.011	0.100
50	DU(*4*)	*2.00*	*2.00*	*1.00*	0.011	0.013	0.009	0.004	0.180	0.114	0.411
	DU(*5*)	*2.50*	*2.92*	*1.17*	0.086	0.123	0.034	0.084	0.522	0.266	0.656
	BB(*5, 1.21, 0.67*)	*3.22*	*2.74*	*0.85*	0.100	0.061	0.522	0.004	0.079	0.634	0.628
	BB(*5, 1.00, 0.67*)	*3.00*	*3.00*	*1.00*	0.031	0.029	0.416	0.033	0.285	0.708	0.805
	BB(*5, 0.84, 0.67*)	*2.79*	*3.20*	*1.15*	0.083	0.115	0.270	0.142	0.582	0.677	0.896
	ZIGP(*3.28, 0.85, 0.04*)	*3.15*	*2.69*	*0.85*	0.101	0.062	0.004	0.003	0.005	0.013	0.034
	ZIGP(*3.28, 0.85, 0.084*)	*3.00*	*3.00*	*1.00*	0.044	0.046	0.014	0.005	0.053	0.062	0.164
	ZIGP(*3.28, 0.85, 0.13*)	*2.85*	*3.28*	*1.15*	0.118	0.148	0.023	0.015	0.193	0.135	0.369
	Poi(*1*)	*1.00*	*1.00*	1.00	0.050	0.051	0.047	0.045	0.004	0.048	0.048
	Poi(*3*)	*3.00*	*3.00*	1.00	0.049	0.048	0.045	0.040	0.017	0.046	0.046
	Poi(*5*)	*5.00*	*5.00*	1.00	0.048	0.049	0.047	0.042	0.023	0.047	0.051
	Bin(*20, 0.15*)	*3.00*	*2.55*	0.85	0.184	0.134	0.006	0.008	0.001	0.008	0.012
	NB(*20, 0.87*)	*3.00*	*3.45*	1.15	0.188	0.215	0.181	0.123	0.061	0.165	0.138
	ZIP(*3.15, 0.048*)	*3.00*	*3.45*	1.15	0.187	0.216	0.064	0.049	0.094	0.064	0.111
	DU(*3*)	*1.50*	*1.25*	0.83	0.147	0.102	0.419	0.000	0.022	0.559	0.544
100	DU(*4*)	*2.00*	*2.00*	1.00	0.012	0.014	0.328	0.048	0.391	0.753	0.912
	DU(*5*)	*2.50*	*2.92*	1.17	0.171	0.212	0.301	0.402	0.840	0.811	0.980
	BB(*5, 1.21, 0.67*)	*3.22*	*2.74*	0.85	0.170	0.130	0.998	0.009	0.138	0.995	0.953
	BB(*5, 1.00, 0.67*)	*3.00*	*3.00*	1.00	0.028	0.028	0.972	0.126	0.530	0.998	0.995
	BB(*5, 0.84, 0.67*)	*2.79*	*3.20*	1.15	0.152	0.182	0.831	0.495	0.877	0.992	1.000
	ZIGP(*3.28, 0.85, 0.04*)	*3.15*	*2.69*	0.85	0.178	0.133	0.062	0.001	0.002	0.069	0.080
	ZIGP(*3.28, 0.85, 0.084*)	*3.00*	*3.00*	1.00	0.046	0.045	0.133	0.003	0.051	0.275	0.417
	ZIGP(*3.28, 0.85, 0.13*)	*2.85*	*3.28*	1.15	0.180	0.207	0.149	0.031	0.266	0.447	0.744

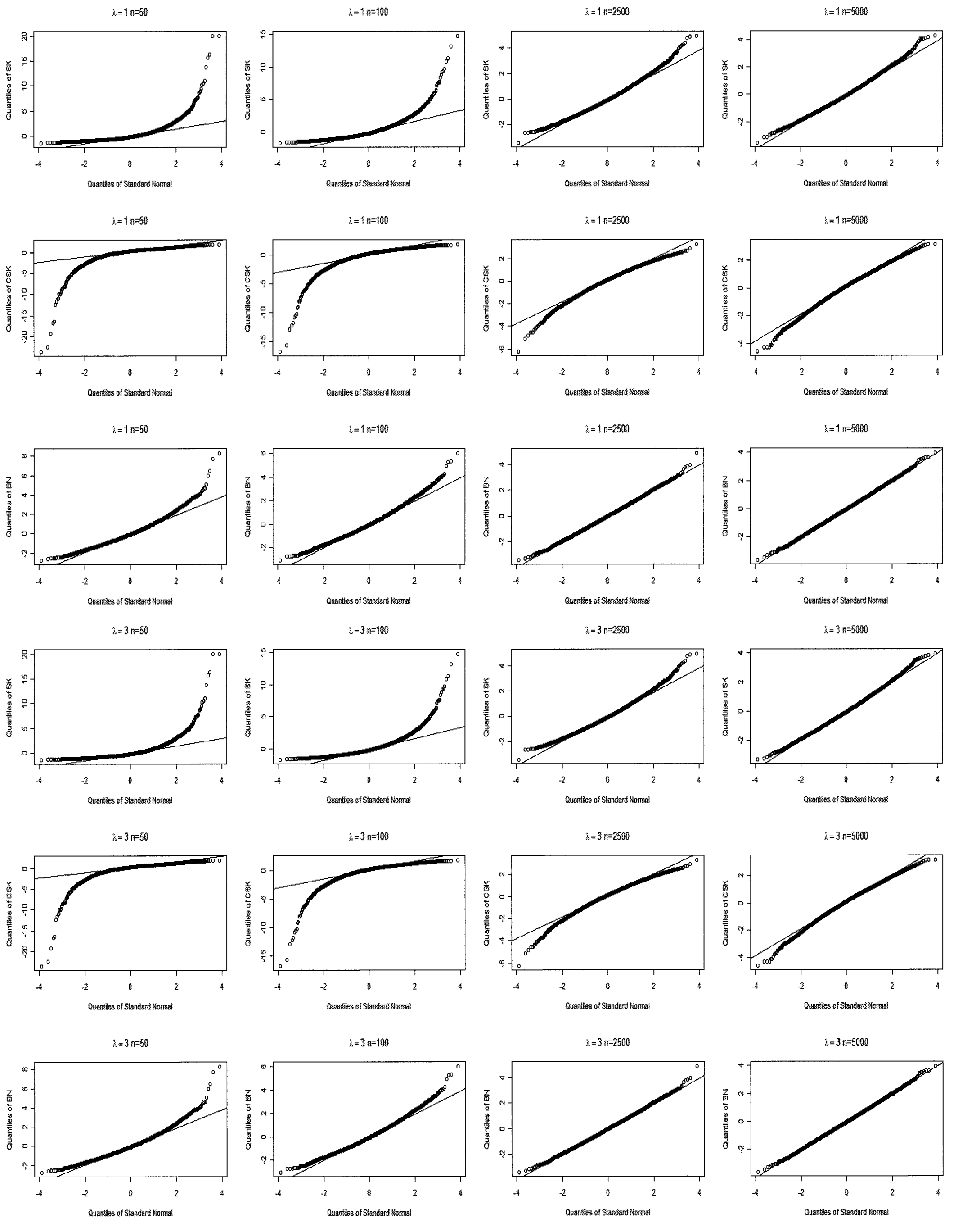

Figure 1. Comparing of the Q-Q plots of simulated values of SK, CSK and BN against standard normal quantiles.

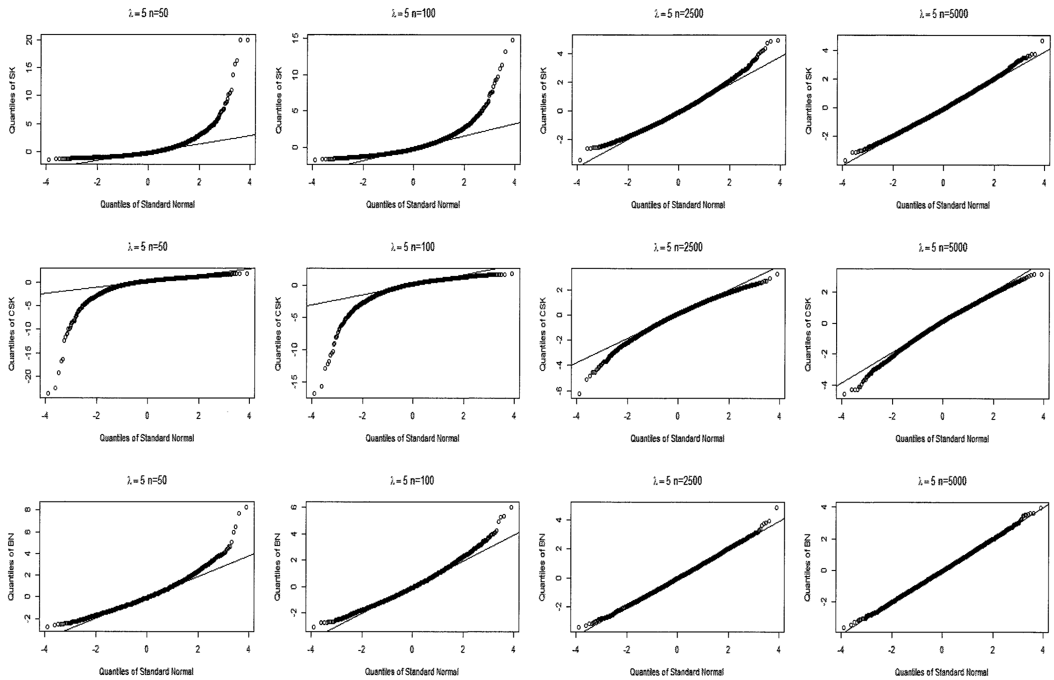

Figure 1. (Continued)

the two tests related to ID. In some situation the CSK test offers a more powerful test than the SK test, and in other cases the power of these two tests are not significantly different.

Therefore, it can be recommended that the CSK test is more likely to be outperformed for detecting a non-Poisson distribution where the variance of the alternative distribution is close to its mean, i.e. $\text{ID} \to 1$. It should be concern with the distributions skewness for testing the fit of sample observations that arise from Poisson distribution. In addition, care must be taken when testing against beta-binomial alternatives since Poisson (λ) approximation to the beta-binomial $\text{BB}\left(r, a, \frac{(r-\lambda)a}{\lambda}\right)$ is accurate when $\frac{\lambda}{r}$ and $\frac{(r-1)\lambda}{ra+\lambda}$ takes on small values [13].

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REFERENCES

[1] Fisher R.A., Thornton H.G. and Mackenzie W.A.,The accuracy of the plating method of estimating the density of bacterial populations, *Ann. Appl. Biol.,* 1922; **9**: 325-359. DOI 10.1111/j.1744-7348.1922.tb05962.x.

[2] Gart J. and Pettigrew H., On the conditional moments of the k-statistics for the Poisson distribution, *Biometrika*, 1970; **57**: 661-664. DOI 10.1093/biomet/57.3.661.

[3] Böhning D., A note on a test for Poisson overdispersion, *Biometrika*, 1994; **81**: 418-419. DOI 10.1093/biomet/81.2.418.

[4] Gupta A.K., Móri T.F. and Székely G.Z., Testing for Poissonity-nomality vs. other infinite divisibility, *Stat. Probab. Lett.*, 1994; **19**: 245-248. DOI 10.1016/0167-7152(94)90111-2.

[5] Karlis D. and Xekalaki E., A simulation comparison of several procedures for testing the Poisson assumption,*The Statistician*, 2000; **49**: 355-382. DOI 10.1111/1467-9884.00240.

[6] Kocherlakota S. and Kocherlakota K., Goodness of fit for discrete distributions, *Commun. Statist. Theor. Meth.*, 1986; **15**: 815-829. DOI 10.1080/03610928608829153.

[7] Gürtler N. and Henze N., Recent and classical goodness-of-fit tests for the Poisson distribution, *J. Stat. Plann. Inference*, 2000; **90**: 207-225. DOI 10.1016/S0378-3758(00)00114-2.

[8] Brown L.D. and Zhao L.H., A Test for the Poisson distribution, *Sankhya Ser. A.*, 2002; **64**: 611-625.

[9] Lehmann E.L. and Casella G.,*Theory of Point Estimation*, 2nd Edn., Springer-Verlag, New York., 1998.

[10] R Development Core Team, *R: A Language and Environment for Statistical Computing*, R Foundation for Statistical Computing, Vienna, Austria, 2009.

[11] Sasom N., Finite-dimesional simple Poisson modules, *Chiang Mai J. Sci.*, 2012; **39**: 678-687. DOI10.1023/A:1007397530551.

[12] Baksh M.F., Böhning D. and Lerdsuwansri R., An extension of an over-dispersion test for count data, *Comput. Stat. Data Anal.*, 2011; **55**: 466-474. DOI 10.1016/j.csda.2010.05.015.

[13] Teerapabolarn K., Poisson aprroximation to the beta binomial distribution, *Int. Math. Forum*, 2009; **25**: 1253-1256.