Modelling Controlled-NOT Gate Based on Rubidium-87 Atom Quantum Bits

Department of Physics and Materials Science, Chiang Mai University
Harit Tunkim

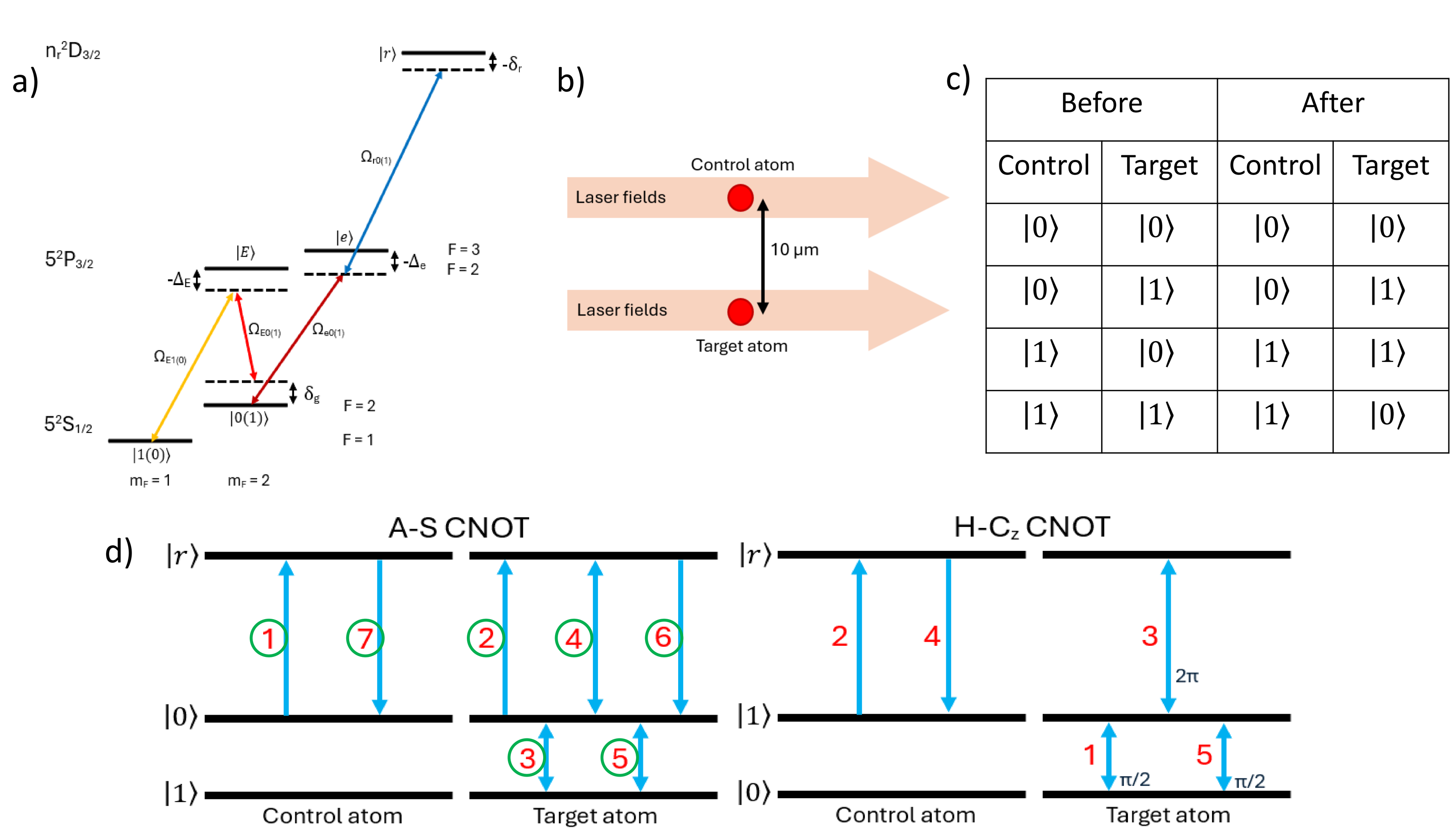
Abstract

The quantum computer with the atom qubit platform harnesses atomic phenomena, such as the electronic state of atoms, atom-electromagnetic field interaction and atom-atom interaction. According to these phenomena, controllability over atomic states using laser fields is crucial in achieving quantum information processing, such as quantum gate operation. The objective is to study Rabi oscillation and Rydberg interaction, then utilize these knowledges to model the Controlled-NOT gate [1]. The study starts by understanding the Rabi oscillation of the rubidium-87 atom modelled by quantum electrodynamics treatment [4]. Then, Quantum Toolbox in Python (QuTiP) [3], utilizing the Lindblad master equation, is used to help with the simulation. Next, the Rydberg interaction between two atoms [5, 6] is included, and then all these theoretical considerations are used to construct a simulation of the Controlled-NOT gate sequence. In this project, 0, 1 and Rydberg state are assigned for the $5S_{1/2}$ F=1, 2 and $97D_{5/2}$. There are four laser fields in this system. Two of them are coupling between 0 and 1 state, and the other two coupling between qubit state and Rydberg state. Results from the simulation show the probability of atoms in each qubit state at the end of sequences and fidelity matrices. The minimum fidelity from the simulation are over 99% for both A-S CNOT and H-C_z CNOT gate sequences depending on the chosen states. In conclusion, this study just builds a simulation for modeling the CNOT gate sequence, and the result will be used as a guideline for real experiments in the future.

Introduction

In the quantum computing, the controllability over atomic state using laser fields is important for quantum gate operation. Hence, this study aim to model and simulate the Controlled-NOT gate sequence by the interaction between 2 Rubidium-87 atoms and laser fields, so the results can be used as a reference for the future work.

Rubidium-87 States & Sequence



Before		After	
Control	Target	Control	Target
|0⟩	|0⟩	|0⟩	|0⟩
|0⟩	|1⟩	|0⟩	|1⟩
|1⟩	|0⟩	|1⟩	|1⟩
|1⟩	|1⟩	|1⟩	|0⟩

Figure 1: a) Rubidium-87 energy diagram of considered states, b) atoms and laser fields alignment and position, c) CNOT gate truth table, d) A-S CNOT and H-C_z CNOT gate sequence driving by π-pulse except for the one that is labeled with π/2 and 2π. It is referenced and adjusted from [1].

QED Treatment for Understanding State Evolution

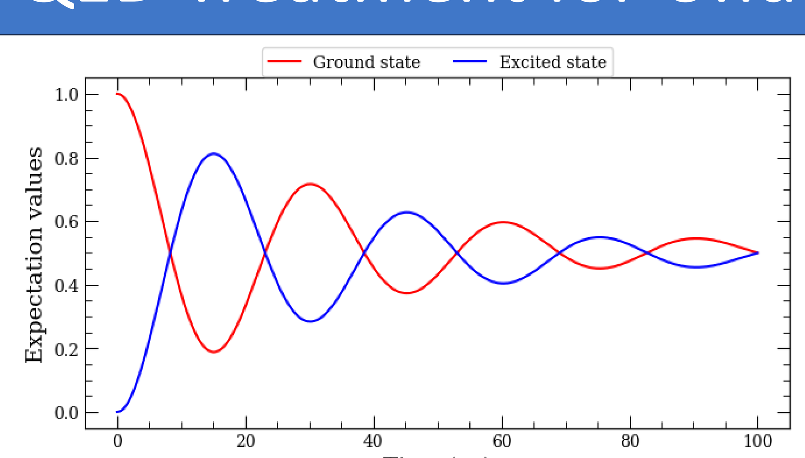


Figure 2: Two-level system Rabi oscillation simulation

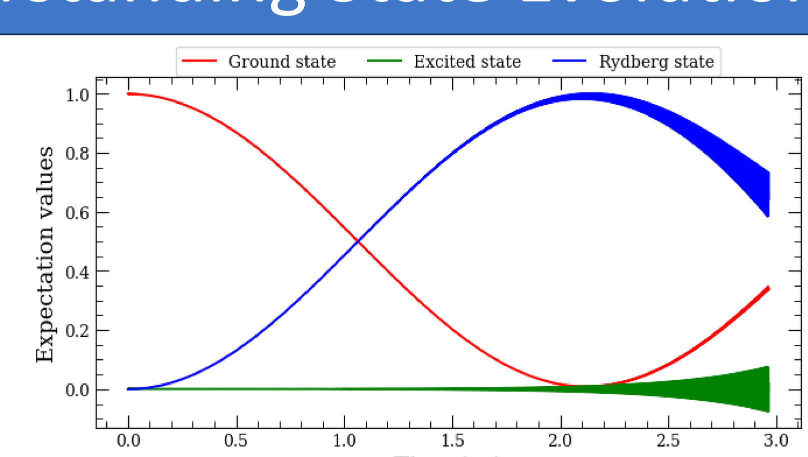


Figure 3: Three-level system Rabi oscillation simulation

To begin with, define the atomic operator by orthonormal basis state $|m\rangle$ and $|n\rangle$ as

$$\sigma = \sum_m \sum_n \sigma_{mn} = \sum_m \sum_n |m\rangle\langle n| \qquad (1)$$

and the density matrix operator as

$$\rho = |\psi\rangle\langle\psi|, \rho_{mn} = \langle m|\rho|n\rangle = \langle\sigma_{nm}\rangle \qquad (2)$$

If m = n, then the operator ρ_{nn} is the probability of atom being in $|n\rangle$ state.

The Hamiltonian

The total Hamiltonian of the system is the combination of 3 Hamiltonian given by

$$H = H_{FA} + H_{FF} + H_I \qquad (3)$$

- The atomic Hamiltonian $\quad H_{FA} = \sum_m \hbar\omega_m |m\rangle\langle m| \qquad (4)$
- The Free Field Hamiltonian $\quad H_{FF} = \hbar\sum_m \omega_\lambda a_\lambda^\dagger a_\lambda \qquad (5)$
- The Interaction Hamiltonian $\quad H_I = \boldsymbol{E}\cdot\boldsymbol{D} \qquad (6)$

where a_λ and $a_\lambda^\dagger$ are annihilation and creation operators respectively that operate on states of photon.

The Heisenberg representation of equation of motion

$$\frac{dO}{dt} = -\frac{i}{\hbar}[O, H] \qquad (7)$$

Using equation (7) on a_λ and $a_\lambda^\dagger$ then σ_{mn}. $\rho_{mn} = \langle\sigma_{nm}\rangle$ can then be written in differential equations form. After that, the simulation will solve these equations numerically, and the result will be shown in the behavior of the atomic state.

QuTiP & Lindblad Master Equation

Quantum Toolbox in Python is open-source software that will be used to simulate the behavior of atomic states [3]. The function that will be used to simulate in this study is the "mesolve" function that implements the Lindblad master equation given by

$$\dot{\rho}(t) = -\frac{i}{\hbar}[H(t),\rho(t)] + \sum_n \left(C_n\rho(t)C_n^\dagger - \frac{1}{2}\{C_n^\dagger C_n, \rho(t)\}\right) \qquad (8)$$

To use this function, the Hamiltonian is transformed into its matrix form. For two levels system, it can be expressed as

$$H = \hbar\begin{pmatrix}0 & \Omega \\ \Omega & \Delta\end{pmatrix} \qquad (9)$$

The collapse operator is given by $C_n = \sqrt{1/\tau}\sigma_{ge}$

where $\Omega = \sqrt{\frac{I_\lambda}{2\hbar^2\varepsilon_0 c}}\hat{\varepsilon}_\lambda\cdot\boldsymbol{D}_{mn}$ is half Rabi frequency,

Δ is the detuning and τ is state lifetime.

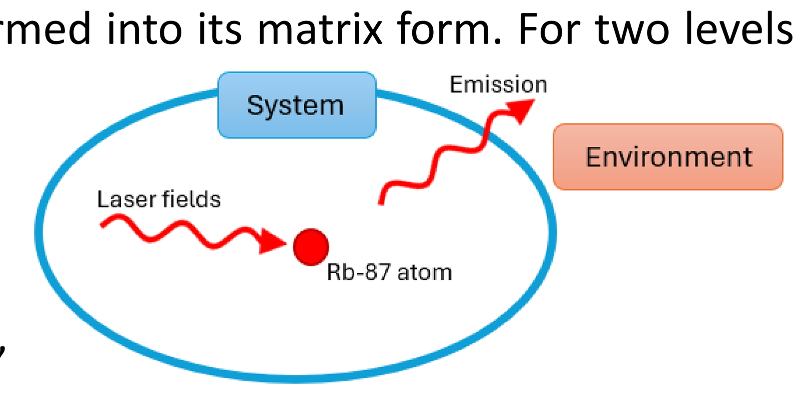


Figure 4: System and Environment definition

Rydberg Blockade

Considering two distinguishable atoms called control, if both atoms are in Rydberg state then there will be dipole-dipole interaction, and the total Hamiltonian will be expressed as [5, 6]

$$H = H_c\otimes I + I\otimes H_t - \frac{C_6}{R^6}|rr\rangle\langle rr| \qquad (10)$$

This interaction result in an energy shift and block the excitation.

CNOT Sequence Simulation

To simulate the Controlled-NOT gate, the Rabi Oscillation and the Rydberg blockage are combined to create each step. The simulation for A-S CNOT gate is shown as

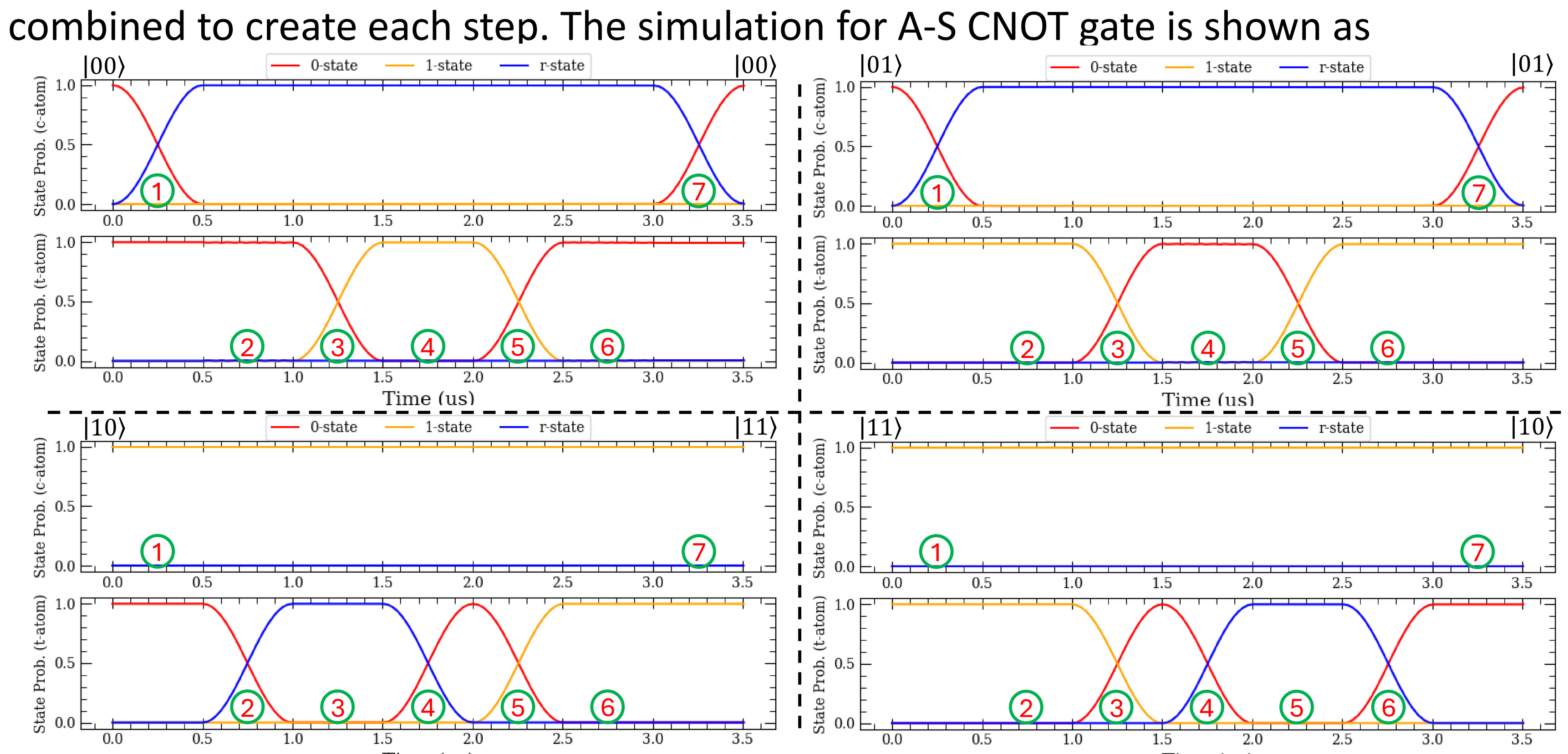


Figure 5: A-S CNOT gate simulation with initial qubit state 00, 01, 10, 11. Number 1 – 7 represent sequence steps from Figure 1d.

The H-C_z CNOT gate simulation can be found in the QR code below.

Conclusion

In conclusion, the result shows the CNOT gate sequence with fidelity of more than 99% for Rydberg state n≥97, with each step duration of 1 μs. We also found that most fidelity loss comes from the detuning and imperfect blockage. In addition, it is more favorable to use a high-n Rydberg state to increase the blockage strength and decrease the error from Rydberg decay. Moreover, if the equipment is precise enough, it is recommended that each step duration be set to be short to decrease decay. Lastly, it must be noted that the collapse operator is approximated, and we still can't find the exact terms as there are many states to be considered.

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Reference & Program