Continuous measurement and control of Nitrogen-Vacancy Center for Magnetic-Field Sensing

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Abstract and Introduction

- We are interested in continuous magnetic-field sensing using Nitrogen-Vacancy (NV) center in diamond
- By considering 3 selected energy levels of NV-Centre as ***Lambda system,*** a steady state called ***Dark state*** can be prepared using ***Coherent population trapping (CPT)*** [1],[2].
- When the NV-Centre interacts with a magnetic field fluctuation the dark state gets perturbed, leaving the CPT stage.
- We theoretically investigate the possibility of using NV-Centre as a magnetic field sensor by **measuring the *population of excited state*** to estimate the value of magnetic field.
- This work explores the population measurement, as an alternative technique to replace the photon counting in the original work [1],[2].

Coherent population trapping (CPT)

System Hamiltonian:

$$\hat{H}_0 = \hbar v_0 |0\rangle\langle 0| + \hbar v_1 |1\rangle\langle 1| + \hbar v_e |e\rangle\langle e|$$

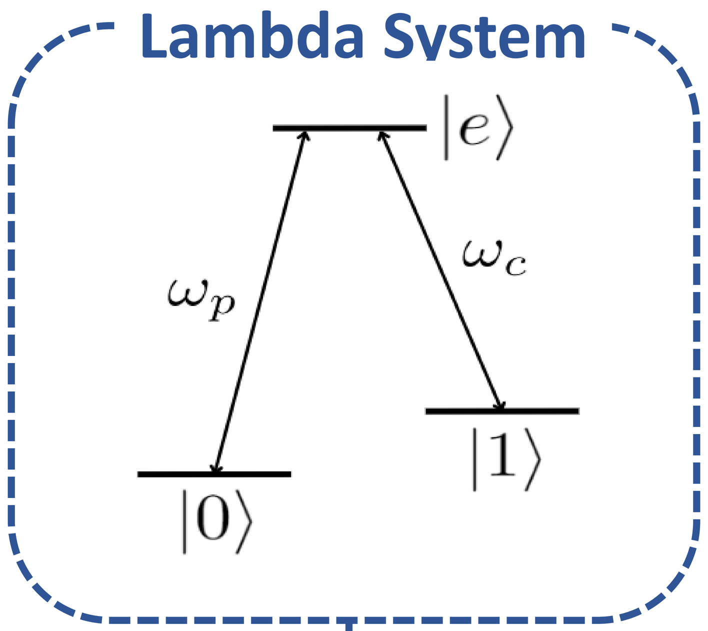
Interaction Hamiltonian:

$$\hat{V} = \frac{\hbar\Omega}{2}\left(e^{i\omega_p t}|0\rangle\langle e| + e^{i\omega_c t}|1\rangle\langle e|\right) + H.c.$$

Solution of $i\hbar|\dot{\psi}\rangle = (\hat{H}_0 + \hat{V})|\psi\rangle$:

$$|\psi\rangle = \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$$

Dark state no population on exited state $|e\rangle$



Lambda system of NV-Centre

From the rotating frame: $H = e^{\frac{i}{\hbar}\hat{H}_0 t}\hat{V}e^{-\frac{i}{\hbar}\hat{H}_0 t}$

Lambda system Hamiltonian in rotating frame:

$$H = \frac{\hbar\Omega}{2}\left(|e\rangle\langle 0| + |e\rangle\langle 1| + |0\rangle\langle e| + |1\rangle\langle e|\right)$$

Prepare an initial state in the dark state using CPT :

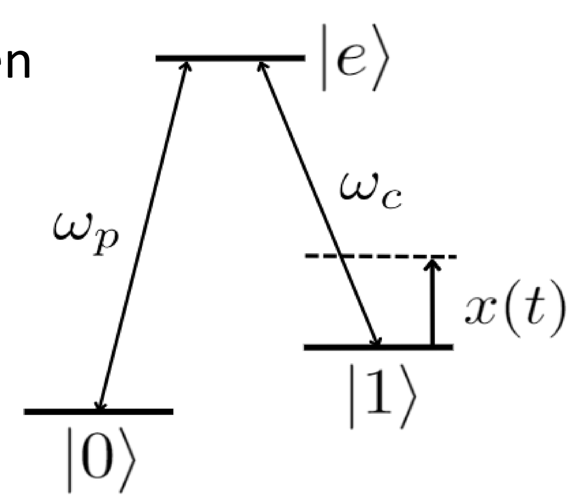
$$\rho_0 = \frac{1}{2}|0\rangle\langle 0| - \frac{1}{2}|0\rangle\langle 1| - \frac{1}{2}|1\rangle\langle 0| + \frac{1}{2}|1\rangle\langle 1|$$

Magnetic field sensing

- Energy level of the Lambda system change when interacting with the magnetic field
- Kicking the state out of the dark state Inducing population on the exited state $|e\rangle$

$$Population\ of\ |e\rangle \neq 0$$

Lambda system



Let us model the **magnetic field fluctuation as an OU process**:

$$dx = -\frac{1}{\tau_N}x\,dt + \sqrt{\frac{2\sigma^2}{\tau_N}}\,dW_t$$

dW_t = Weiner increment
τ_N = memory time

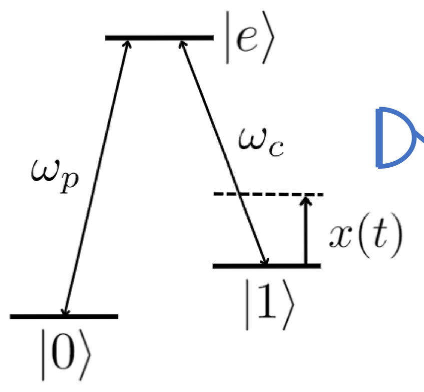
Lambda system Hamiltonian in rotating frame:

$$H = \frac{\hbar\Omega}{2}\left(|e\rangle\langle 0| + |e\rangle\langle 1| + |0\rangle\langle e| + |1\rangle\langle e|\right) + \hbar[\Delta_0 + x(t)]|1\rangle\langle 1|$$

Δ_0= Raman detuning

Jump Measurement

- We measure the exited state population to infer to magnetic field



Photon count → $y_{readout}$ →(estimation) $x(t)$

$\hat{c}_1 = |0\rangle\langle e|$
$\hat{c}_2 = |0\rangle\langle e|$

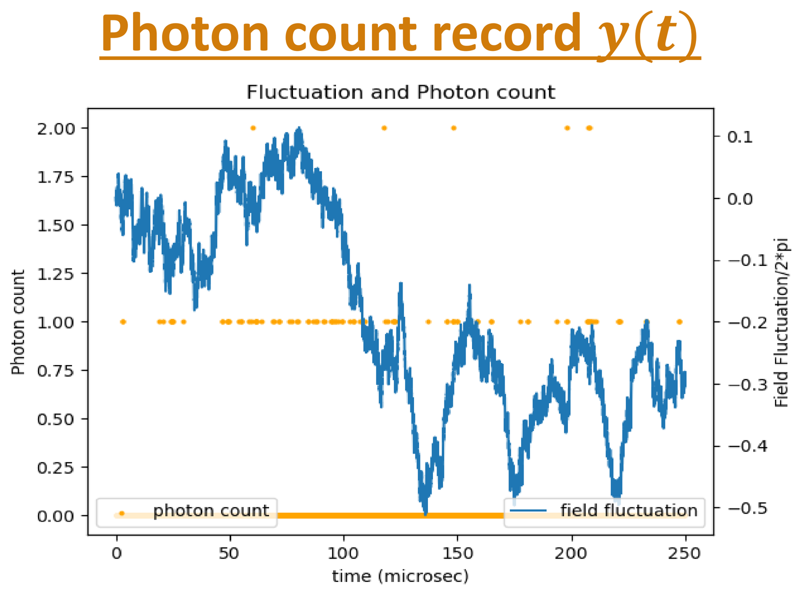
Estimated magnetic field

Quantum jump photon detection:

Stochatic Schrodinger's equation

$$d|\psi(t)\rangle = -i\left(\hat{H} - \frac{i\Gamma}{2}|e\rangle\langle e|\right)|\psi(t)\rangle + \frac{\Gamma}{2}\langle|e\rangle\langle e|\rangle|\psi\rangle dt + \left(\frac{|0\rangle\langle e|}{\sqrt{\langle|e\rangle\langle e|\rangle|\psi\rangle}} - I\right)|\psi(t)\rangle dN_0(t) + \left(\frac{|1\rangle\langle e|}{\sqrt{\langle|e\rangle\langle e|\rangle|\psi\rangle}} - I\right)|\psi(t)\rangle dN_1(t)$$

Photon count record $y(t)$



Bayesian estimator

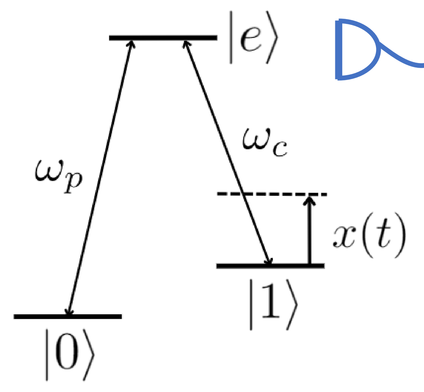
$$P(x_n|y_n,\dots,y_1) \propto \int P(y_n|x_n)P(x_n|x_{n-1})P(x_{n-1}|y_{n-1},\dots,y_1)dx_{n-1}$$

$$\tilde{x}_n = \int x_n P(x_n|y_n,\dots,y_{n-1})dx_n$$

Estimated magnetic field value at time $t_n = ndt$

Diffusive Measurement

- We measure the exited state population to infer to magnetic field



Population measurement → $y_{readout}$ →(estimation) $x(t)$

$\hat{c} = \sqrt{\gamma/2}|e\rangle\langle e|$

Estimated magnetic field

Readout y from the exited state population measurement:

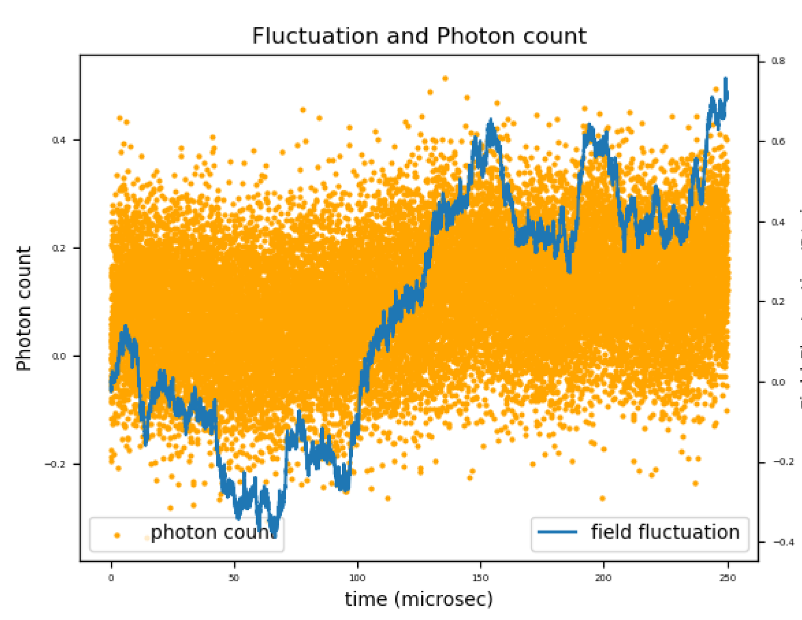
$$y_{readout} = Tr[(\hat{c} + \hat{c}^+)\rho] + \frac{dW_t}{dt} = Tr[\sqrt{2\gamma}|e\rangle\langle e|] + \frac{dW_t}{dt}$$

Measurement operator: $\hat{M} = I - (1/2)\hat{c}^+\hat{c}dt + \hat{c}ydt$

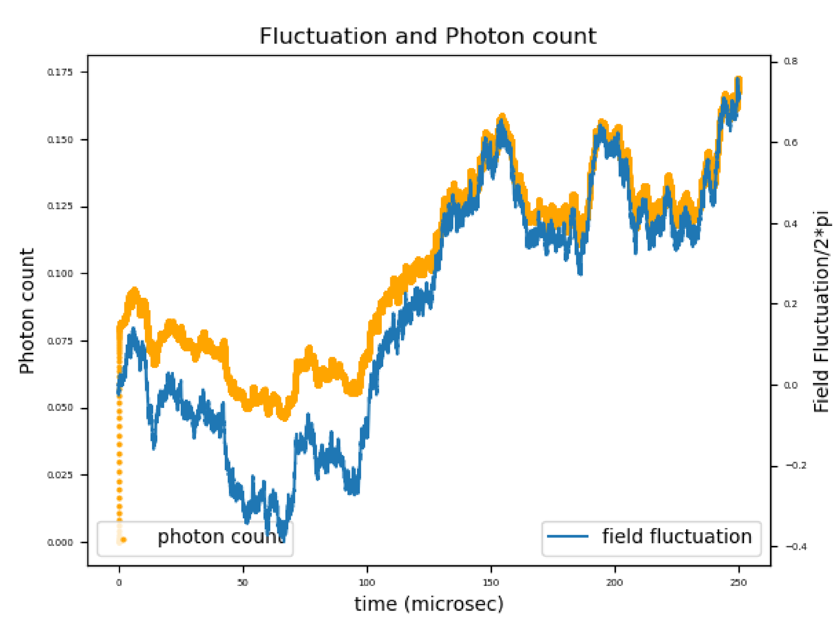
Measurement evolution:
Where $\hat{U} = e^{-iHdt}$

$$\dot{\rho}(t) = \frac{\hat{M}\hat{U}\rho(t)\hat{U}^+\hat{M}^+}{Tr[\hat{M}\hat{U}\rho(t)\hat{U}^+\hat{M}^+]}$$

Measurement readout record $y(t)$



Readout without noise



Conclusion

- The recreated simulation of photon counts matches with [1]
- The diffusive readout after noise removal shows a trend similar to the fluctuation
- The estimation part is still in progress

[1] Shu-Hao Wu , Ethan Turner, and Hailin Wang, "Continuous real-time sensing with a nitrogen-vacancy center via coherent population trapping" (2021)
[2] Ethan Turner, Shu-Hao Wu, Xinzhu Li, and Hailin Wang "Real-time magnetometry with coherent population trapping in a nitrogen-vacancy center" (2022)