An Algorithm for Finding Gold Derivatives Prices from Price Data

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ABSTRACT

In this independent study, we are interested in gold derivatives. The historical data are collected from www.investing.com. There are three objectives to be achieved in this study. First, we estimate the volatility of gold prices by using two methods; namely, the standard deviation of returns and GARCH(1,1) model. After obtaining the volatilities from the two methods, we apply the Black-Sholes-Merton formula to calculate the options prices. Finally, we compare the results from these methods.

DERIVATIVES

A derivative can be defined as a financial contract whose value depends on the underlying asset. The variable underlying assets are often the prices of traded assets. The "buyer" and the "seller" agree today on the quantity, price, and settlement date of the underlying asset. Derivatives are traded on the futures market, such as the Thailand Futures Exchange (TFEX). Importantly, derivatives have a limited lifespan, such as 1 month, 3 months, or 6 months, so it is essential to regularly monitor the contract's expiration date. We are focusing on two main types derivatives those are futures and options.

FUTURES

Futures is a contract which the buyer and seller agree to trade an asset at a predetermined price today, with delivery and payment in the future. The buyer and seller are not required to hold the contract until maturity and they have to buy or sell that agreed upon the contract.

OPTIONS

For options, it is similar to a futures in that it is an agreement between two parties to buy or sell an asset at a predetermined future date for a specified price. The key difference between options and futures is that the buyer is not obliged to exercise their agreement to buy or sell. It is an opportunity only and the buyer must pay a premium.

call options

It gives the holder the right to buy the underlying asset. It always be exercise at the expiration date if the stock price is above the strike price. Focusing on increasing of stock price.

put options

Whereas put option gives the holder the right to sell the underlying asset. Focusing on decreasing of stock price.

BLACK-SCHOLES-MERTON MODEL

$$c = S_0N(d_1) - Ke^{-rT}N(d_2) \qquad p = Ke^{-rT}N(-d_2) - S_0N(-d_1)$$

Where

$$d_1 = \frac{ln(S_0/K) + (r + \sigma^2/2)T}{\sigma\sqrt{T}}$$

$$d_2 = \frac{ln(S_0/K) + (r - \sigma^2/2)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}$$

- S_0 = current stock price
- K = strike price
- T = time to expiration
- σ = volatility
- r = risk-free interest rate

$$\text{Volatility per year} = \text{Volatility per trading day} \times \sqrt{\text{Number of trading days per year}}$$

$$T = \frac{\text{Number of trading days until option maturity}}{252}$$

THE STANDARD DEVIATION OF RETURN

$$\sigma_n^2 = \frac{1}{m}\sum_{i=1}^{m} u_{n-i}^2$$

$$u_i = \frac{S_i - S_{i-1}}{S_{i-1}}$$

Where u_i is the return in the market between the end of day $i-1$ and the end of day i

THE GARCH(1,1) MODEL

$$\sigma_n^2 = \omega + \alpha u_{n-1}^2 + \beta\sigma_{n-1}^2$$

To know ω, α, and β. We use maximum likelihood method. Define $v_i = \sigma_i^2$.

$$\sum_{i=1}^{m}\left[-ln(v_i) - \frac{u_i^2}{v_i}\right]$$

RESULT

THE STANDARD DEVIATION OF RETURN

	A	B	C	D	E	F	G	H	I
1	Date	t	Price	Return (ui)	ui^2				
2	01/06/2025	23	2,647.40						
3	01/07/2025	22	2,665.40	0.006799	0.00004623		volatility per	0.0094953	0.00009016
4	01/08/2025	21	2,672.40	0.002626	0.00000690				
5	01/09/2025	20	2,690.80	0.006885	0.00004741		annual volati	0.15073316	
6	01/10/2025	19	2,715.00	0.008994	0.00008088				
7	01/13/2025	18	2,678.60	-0.013407	0.00017975				
8	01/14/2025	17	2,682.30	0.001381	0.00000191				
9	01/15/2025	16	2,717.80	0.013235	0.00017516				
10	01/16/2025	15	2,750.90	0.012179	0.00014833				
11	01/17/2025	14	2,748.70	-0.000800	0.00000064				
12	01/21/2025	13	2,785.60	0.013425	0.00018022				
13	01/22/2025	12	2,797.40	0.004236	0.00001794				
14	01/23/2025	11	2,778.30	-0.006828	0.00004662				
15	01/24/2025	10	2,792.70	0.005183	0.00002686				
16	01/27/2025	9	2,752.40	-0.014430	0.00020824				
17	01/28/2025	8	2,781.00	0.010391	0.00010797				
18	01/29/2025	7	2,781.50	0.000180	0.00000003				
19	01/30/2025	6	2,845.20	0.022901	0.00052447				
20	01/31/2025	5	2,835.00	-0.003585	0.00001285				

Volatility

1	0.1507
6	0.1526
12	0.1545

THE GARCH(1,1) MODEL

1	6	12
0.1582	0.1534	0.1570

	A	B	C	D	E	F	G	H	I	J
1	Date	t	Price	Return (ui)	variance rate (vi)	likelihood		parameter		
2	01/06/2025	23	2,647.40					omega	0.00004836	
3	01/07/2025	22	2,665.40	0.00679912				beta	0.51363948	
4	01/08/2025	21	2,672.40	0.00262625	0.00004623	9.832724267		alpha	0.00000000	
5	01/09/2025	20	2,690.80	0.0068852	0.00007211	8.879923709				
6	01/10/2025	19	2,715.00	0.00899361	0.00008540	8.421034465		sum	174.8537918	
7	01/13/2025	18	2,678.60	-0.013407	0.00009223	7.342276127				
8	01/14/2025	17	2,682.30	0.00138132	0.00009573	9.234020525		long-run volatility	0.009971765	0.00009944
9	01/15/2025	16	2,717.80	0.01323491	0.00009753	7.43939249				
10	01/16/2025	15	2,750.90	0.01217897	0.00009846	7.719382843		annual volatility	0.158296869	
11	01/17/2025	14	2,748.70	-0.0007997	0.00009893	9.214590442				
12	01/21/2025	13	2,785.60	0.01342453	0.00009918	7.401480845				
13	01/22/2025	12	2,797.40	0.00423607	0.00009930	9.036626501				
14	01/23/2025	11	2,778.30	-0.0068278	0.00009937	8.747530638				
15	01/24/2025	10	2,792.70	0.00518303	0.00009940	8.946090729				
16	01/27/2025	9	2,752.40	-0.0144305	0.00009942	7.121601353				
17	01/28/2025	8	2,781.00	0.01039093	0.00009943	8.130149752				
18	01/29/2025	7	2,781.50	0.00017979	0.00009943	9.215717783				
19	01/30/2025	6	2,845.20	0.02290131	0.00009943	3.941447439				
20	01/31/2025	5	2,835.00	-0.003585	0.00009943	9.086756186				

CALL OPTIONS PRICE

t	Date	call options price	S_0	T	standard deviation of return call options price cal			GARCH(1,1) call options price cal		
					1 month	6 months	12 months	1 month	6 months	12 months
					0.1513	0.1539	0.1554	0.1583	0.1534	0.1570
1	06/02/2025	22.10	2876.70	0.0873	36.73	37.59	38.08	39.04	37.42	38.61
2	07/02/2025	25.60	2,887.60	0.0833	39.99	40.84	41.33	42.29	40.68	41.86
3	10/02/2025	44.70	2,934.40	0.0714	59.40	60.19	60.65	61.54	60.04	61.14
4	11/02/2025	31.00	2,932.60	0.0675	56.78	57.56	58.01	58.87	57.41	58.48
5	12/02/2025	42.30	2,928.70	0.0635	52.95	53.70	54.14	54.98	53.56	54.60
6	13/02/2025	47.00	2,945.40	0.0595	61.23	61.94	62.35	63.14	61.81	62.79
7	14/02/2025	19.90	2,900.70	0.0556	35.27	35.98	36.38	37.17	35.84	36.82
8	18/02/2025	26.80	2,949.00	0.0397	54.60	55.16	55.49	56.12	55.06	55.84
9	19/02/2025	36.00	2,936.10	0.0357	44.49	45.05	45.37	45.99	44.94	45.71
10	20/02/2025	43.30	2,956.10	0.0317	55.40	55.88	56.15	56.69	55.78	56.45
exp :	Mar-25			MSE :	284.51	307.36	321.03	348.44	302.89	335.99
r	0.0401									
K	2920									

standard deviation of return					
1 month		6 months		12 months	
d1	d2	d1	d2	d1	d2
-0.2335	-0.2782	-0.2288	-0.2743	-0.2262	-0.2721
-0.1571	-0.2008	-0.1537	-0.1981	-0.1518	-0.1967
0.2127	0.1723	0.2098	0.1687	0.2082	0.1666
0.1981	0.1588	0.1954	0.1554	0.1939	0.1535
0.1639	0.1258	0.1618	0.1230	0.1606	0.1214
0.3177	0.2808	0.3130	0.2755	0.3104	0.2724
-0.1057	-0.1413	-0.1033	-0.1395	-0.1019	-0.1385
0.3958	0.3656	0.3896	0.3589	0.3861	0.3552
0.2567	0.2281	0.2528	0.2238	0.2507	0.2213
0.5165	0.4895	0.5082	0.4808	0.5036	0.4759

GARCH(1,1)					
1 month		6 months		12 months	
d1	d2	d1	d2	d1	d2
-0.2212	-0.2680	-0.2297	-0.2750	-0.2234	-0.2698
-0.1482	-0.1939	-0.1544	-0.1986	-0.1498	-0.1951
0.2051	0.1628	0.2104	0.1694	0.2065	0.1645
0.1911	0.1500	0.1959	0.1560	0.1923	0.1515
0.1584	0.1185	0.1622	0.1235	0.1593	0.1198
0.3054	0.2667	0.3139	0.2765	0.3076	0.2693
-0.0994	-0.1367	-0.1037	-0.1399	-0.1005	-0.1375
0.3796	0.3481	0.3908	0.3602	0.3825	0.3512
0.2466	0.2167	0.2536	0.2246	0.2484	0.2188
0.4949	0.4667	0.5098	0.4825	0.4987	0.4708

CONCLUSION

In this study, from the historical data of the gold futures prices, we obtain the volatility from two methods, namely the standard deviation of return and the GARCH(1,1) model. For the standard deviation of return, we get σ = 0.1507, σ = 0.1526 and σ = 0.1545 from 1 month, 6 months and 12 months of data, respectively. And for the GARCH(1,1) model, we get σ = 0.1582, σ = 0.1534 and σ = 0.1570 from 1 month, 6 months and 12 months of data, respectively. Then, we calculate each call options price with difference volatility and use mean square error for compare which volatility from difference method and numbers of data is better. And it is 1 month of data and using standard deviation of return which has MSE equal to 284.51, the best value of MSE. It can be said that the less data that are used to calculate volatility is the less error.

REFERENCES

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[2] https://www.investopedia.com/articles/economics/09/why-gold-matters.asp (January 12, 2023)