The Dihedrant of Lanna Magic Squares

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Abstract

In this independent study, we investigated the dihedrant of the pandiagonal Lanna magic squares, which is a 4x4 matrix. By using knowledge of dihedral subgroups, we found that the dihedrant of all pandiagonal Lanna magic squares has only possible values: 512 or -512.

Introduction

The Lanna magic square, numerical arrangement found in Northern Thai yantras that incorporates Lanna characters and numerals. These numberals were converted into Hindu-Arabic numeral, and the entire square was subsequently reduced and the magic constant of the square is 32.

Preliminary

Definition 1: The Lanna Magic square is a specific type of 4x4 magic square that includes integers from 1 to 15, with the number 8 appearing twice. This arrangement ensures that the sum of the numbers in each row, in each column, and the two principal main diagonals are identical, creating a "magic constant."

Definition 2: A 4x4 of pandiagonal Lanna magic square.

a	b	c	d
e	f	g	h
i	j	k	l
m	n	o	p

The sum of the numbers **in each row** : a+b+c+d=e+f+g+h=i+j+k+l=m+n+o+p=M

The sum of the numbers **in each column** : a+e+i+m=b+f+j+n=c+g+k+o=d+h+l+p=M

The sum of the numbers **in each of the two principal main diagonals** :
a+f+k+p=d+g+j+m=M

The sum of the number **in the six additional diagonals** :
a+h+k+n=b+e+l+o=c+f+i+p=M and d+e+j+o=c+h+i+n=b+g+l+m=M

Examble 3:

Let $L_0 =$

2	7	10	13
8	15	4	5
8	1	12	11
14	9	6	3

, then L_0 is a pandiagonal Lanna magic square with M=32.

Definition 4: Five types of mathematical transformations

T_1 is the reflection about the diagonal.

T_2 is the rotation through 90 degree counter-clockwise.

T_3 is the move of the first column to the last column.

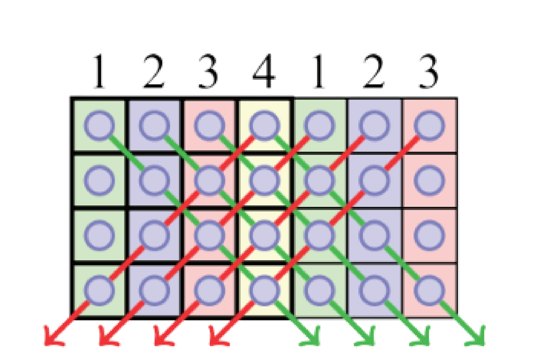
T_4 is the move of the first row to the last row.

T_5 is the transformation of the pandiagonal Lanna magic square.

Definition 5: A permutation is even if its total number of inversion is even, and odd if its total number of inversions is odd.

Definition 6: For any 4x4 matrix A, the dihedrant of A is defined as :

$$dih(A)=\sum_{\sigma\in D_4} sig(\sigma)\prod_{i=1}^{4} a_{i,\sigma(i)}$$



From Example 3, we have $dih(L_0)=512$.

Objectives

1. To study the key properties of the dihedrant.
2. To study the important properties of the pandiagonal Lanna magic square.
3. To explore the medthod for determining the dihedrant value of the pandiagonal Lanna magic square.

Results

Dihedrant Value of pandiagonal Lanna magic square

Theorem 1. If L is pandiagonal Lanna magic square, then $dih(T_1L)=dih(L)$

Theorem 2. If L is pandiagonal Lanna magic square, then $dih(T_2L)=-dih(L)$

Theorem 3. If L is pandiagonal Lanna magic square, then $dih(T_3L)=dih(L)$

Theorem 4. If L is pandiagonal Lanna magic square, then $dih(T_4L)=dih(L)$

Theorem 5. If L_0 is pandiagonal Lanna magic square as in Example 3, then $dih(T_5L_0)=-dih(L_0)$

Theorem 6.

Let L_0 is pandiagonal Lanna magic square as in Example 3, then it can be expressed in the form:

$$\mathrm{P}=\{T_1{}^aT_2{}^bT_3{}^cT_4{}^dT_5{}^e(L_0):a,b,c,d,e\geq 0\}$$

where $|P|=384$.

If $L\in\mathrm{P}$, then

$$L=T_1^{a_1}T_2^{a_2}T_3^{a_3}T_4^{a_4}T_5^{a_5}(L_0)$$

and the dihedral index is determined as follows:

$$\mathrm{dih}(L)=\begin{cases}512, & a_2+a_5 \text{ is even}\\ -512, & a_2+a_5 \text{ is odd}\end{cases}.$$

References

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