



Analysis of playing the 2048 game

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Abstract

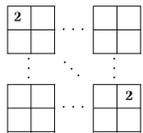
In this independent study, we study about the analysis of game 2048 Finding the minimum steps required to reach the largest possible number on 2 x 2, 3 x 3 and 4 x 4 sized boards, using mathematical methods and knowledge for the analysis and program MATLAB.

Research Objective

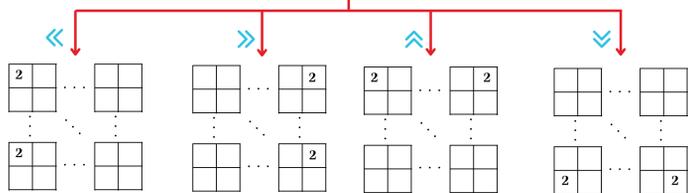
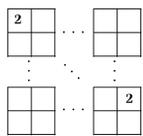
1. To study and understand the gameplay mechanics of 2048.
2. To analyze the largest possible number that can be achieved in 2048 on 2 x 2, 3 x 3 and 4 x 4 sized boards.
3. To determine the minimum steps required to reach the largest possible number in 2048 on 2 x 2, 3 x 3 and 4 x 4 sized boards

2048 Game

1. Before the game starts, two random tiles with the number 2 will appear on an n x n sized boards.

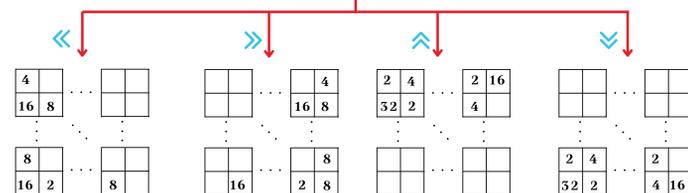
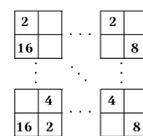


2. The player must slide the tiles in one of four directions: Left, Right, Up, or Down. After sliding, all tiles on the grid will move in the chosen direction until they either collide with another tile or reach the edge of the grid.

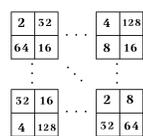


3. With each move, a new tile with the number 2 will randomly appear in an empty space on the grid.

4. When two tiles with the same number collide, they merge into a single tile with a value equal to their sum.



5. The game ends when there are no empty spaces left on the grid and no possible moves remain.



Reference

- [1] Chalermpong. (2012). Mathematical Induction. The website can be accessed from <https://shorturl.asia/gNaSI>
- [2] Kullanat. (2023). Combination. The website can be accessed from <https://tuemaster.com/blog/combination/>
- [3] Chaiporn Thangthong. Introductio to Basic MATLAB Programming, Mathematical package, Department of Mathermatics (2021)

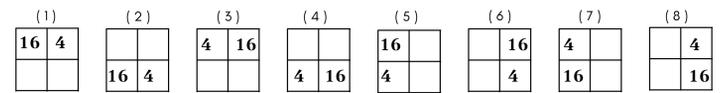
Interested 2048 game

To determine the minimum steps required to reach the largest possible number, we categorize the actions as follows:

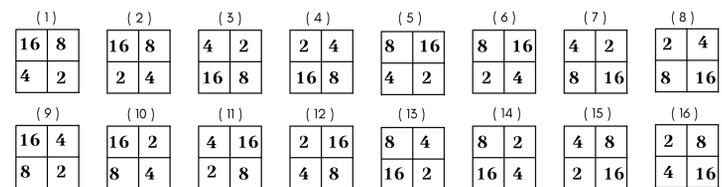
- **Step:** Placing a single "2" in an empty space after a move.
- **Slide:** Sliding in one of the four directions.

The key scenarios to consider are as follows:

Slide 9 It will be a shift that results in the number 16, as shown below



Step 14 By adding the number 2 in every case from the 13th slide, no further shifts can combine any numbers, as shown below



This implies that, in a 2 x 2 sized boards, the laigest possible number is 16, and the minimum steps required to reach the largest possible number is 9.

Mains Result

Table 1: Number of moves required to generate different numbers on an 2 x 2 sized boards.

different numbers	Number of moves
$2^2 = 4$	1
$2^3 = 8$	4
$2^4 = 16$	9

Table 2: Number of moves required to generate different numbers on an 3 x 3 sized boards.

different numbers	Number of moves
$2^2 = 4$	1
$2^3 = 8$	4
$2^4 = 16$	9
$2^5 = 32$	18
$2^6 = 64$	35
$2^7 = 128$	68
$2^8 = 256$	133
$2^9 = 512$	262

From Table 2, the following correlation can be observed:

- 4 is generated after 1 moves
- 8 is generated after $(1 \times 2) - (-2) = 4$ moves
- 16 is generated after $(4 \times 2) - (-1) = 9$ moves
- 32 is generated after $(9 \times 2) - 0 = 18$ moves
- 64 is generated after $(18 \times 2) - 1 = 35$ moves
- 128 is generated after $(35 \times 2) - 2 = 68$ moves
- 256 is generated after $(68 \times 2) - 3 = 133$ moves
- 512 is generated after $(133 \times 2) - 4 = 262$ moves

Theorem 1 The largest number possible for an n x n sized board is 2^{n^2} .

Theorem 2 Minimum steps required in the 2048 game on an n x n sized board to Generate the Number 2^n is $2^{n-1} + n - 3$ for $n \geq 2$.

Conclusion

Table 3: The table below shows largest possible number and the minimum steps required to reach the largest possible number on 2 x 2, 3 x 3 and 4 x 4 sized boards.

	2x2	3x3	4x4
largest possible number	$2^4 = 16$	$2^9 = 512$	$2^{16} = 65,536$
minimum steps required	9	262	32,781

Qr code for viewing MATLAB code

