Some properties regarding Fibonacci polynomial with integer coefficients modulo 2

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Abstract

In this independent study, we have studied some properties of Fibonacci polynomials by considering the case where the coefficients of the polynomials are integers modulo 2. Additionally, we have explored the patterns of the Fibonacci polynomials under these conditions, including their structural properties and the nature of the coefficients. We also examined the properties that we hypothesized.

Introduction

The Fibonacci sequence is one of the most famous number sequences in history, created by the Italian mathematician Leonardo of Pisa, also known as Leonardo Fibonacci, or simply Fibonacci.

Fibonacci polynomials are a sequence of polynomials that can be considered as a general form of Fibonacci numbers.

$$F_n(x) = \begin{cases} 0 & \text{if } n = 0 \\ 1 & \text{if } n = 1 \\ xF_{n-1}(x) + F_{n-2}(x) & \text{if } n \geq 2 \end{cases}$$

In this independent research, we focus on studying Fibonacci polynomials in the case where the coefficients of the polynomials are considered modulo 2. This is an important concept in number theory and discrete mathematics. Studying polynomials modulo 2 can lead to a deeper understanding of the structure and properties of Fibonacci polynomials under these conditions.

Objectives

To study some properties regarding Fibonacci polynomials with integer coefficients modulo 2

Reference

- Benjamin Arthur, Combinatorial enumeration problems, Proofs that Really Count The Art of Combinatorial Proof, [Washington, DC]:Mathematical Association of America, 2003: page 141

Results

Definition 3.1 Let $\overline{F_n(x)}$ be a polynomial obtained by taking the coefficients of each term of $\overline{F_n(x)}$ modulo 2, for all $n \in \{0, 1, 2, 3, \ldots\}$.

Proposition 3.2

$$\overline{F_n(x)} = \begin{cases} 0 & \text{if } n = 0 \\ 1 & \text{if } n = 1 \\ x\overline{F_{n-1}(x)} + \overline{F_{n-2}(x)} & \text{if } n \geq 2 \end{cases}$$

where $n \in \{2, 3, 4, \ldots\}$

Theorem 3.3 $\overline{F_{2k-1}}(x)$ will always have a term with a coefficient of 1.

Theorem 3.4 $\overline{F_{4k-2}}(x)$ will always have a term with x.

Theorem 3.5 $\overline{F_{8k-5}}(x)$ and $\overline{F_{8k-3}}(x)$ will always have a term with x^2.

Theorem 3.6 $\overline{F_{8k-4}}(x)$ will always have a term with x^3.

Theorem 3.7 $\overline{F_{m+n}}(x) = \overline{F_{m+1}}(x)\overline{F_n}(x) + \overline{F_m}(x)\overline{F_{n-1}}(x)$.

Corollary 3.8 $\overline{F_{2n}}(x) = \overline{F_n}(x)(\overline{F_{n+1}}(x) + \overline{F_{n-1}}(x))$ or $\overline{F_{2n}}(x) = x\overline{F_n}(x)^2$.

Theorem 3.9 $\overline{F_{2^k}}(x) = x^{2^k-1}$

Conclusion

In this independent research, we have studied some properties of Fibonacci polynomials with integer coefficients modulo 2. The results of the study on some properties of Fibonacci polynomials with integer coefficients modulo 2 are as follows:

$\overline{F_{8k-7}(x)}$	$\overline{F_{8k-6}(x)}$	$\overline{F_{8k-5}(x)}$	$\overline{F_{8k-4}(x)}$	$\overline{F_{8k-3}(x)}$	$\overline{F_{8k-2}(x)}$	$\overline{F_{8k-1}(x)}$	$\overline{F_{8k}(x)}$
1		1		1		1	
	x				x		
		x^2		x^2			
			x^3				

From the table, the following conclusions can be drawn:

- If the remainder when divided by 8 is 1, 3, 5, or 7, there will always be a term with a coefficient of 1.
- If the remainder when divided by 8 is 2 or 6, there will always be a term with x.
- If the remainder when divided by 8 is 3 or 5, there will always be a term with x^2.
- If the remainder when divided by 8 is 4, there will always be a term with x^3.

And the properties $\overline{F_{2^k}}(x) = x^{2^k-1}$.