AN EFFICIENCY COMPARISON OF TESTING THE HOMOGINIETY OF VARIANCE IN TWO POPULATIONS USING NONPARAMETRIC STATISTICS

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ABSTRACT

The objective of this research is to compare the efficiency of the Fligner-Killeen test, Ansari-Bradley test, and Moses test for testing homogeneity of variances between two populations, considering their ability to control the probability of type I error and the power of the test under normal, beta, and Weibull distributions. Sample sizes were (10,10), (20,20), (30,30), (20,10), (30,15), and (40,20), with variance ratios of 1, 1.5, 2.25, and 3.375 at a significance level of 0.05, based on 40,000 simulations for each scenario.

The simulation results indicated that all tests were able to control the probability of type I error in all cases. In terms of power, Fligner-Killeen test generally provided the highest power when the data were normal distribution, whereas Ansari-Bradley test performed best when sample sizes were small. For Weibull distribution, Fligner-Killeen test yielded the highest power in all cases. For beta distribution, Ansari-Bradley test provided higher power for equal sample sizes, while Fligner-Killeen test generally performed better for unequal sample sizes, particularly when the sample sizes varied inversely with the variance ratio. When the sample sizes varied directly with the variance ratio, Ansari-Bradley test provided the highest power in almost all situations.

INTRODUCTION

Testing for variance is an important step in statistical data analysis. however, in practice, data often do not follow a normal distribution, which may lead parametric methods to produce misleading results. Consequently, nonparametric methods are employed as they provide reliable results even when underlying assumptions are violated.

This study compares the performance of three non-parametric statistical methods: Fligner-Killeen test, Ansari-Bradley test, and Moses test to compare their performance in testing homogeneity of variances under various conditions. The comparison is based on Type I error rates and power of a test. The research results provide guidance in selecting the most appropriate method for obtaining reliable analytical results.

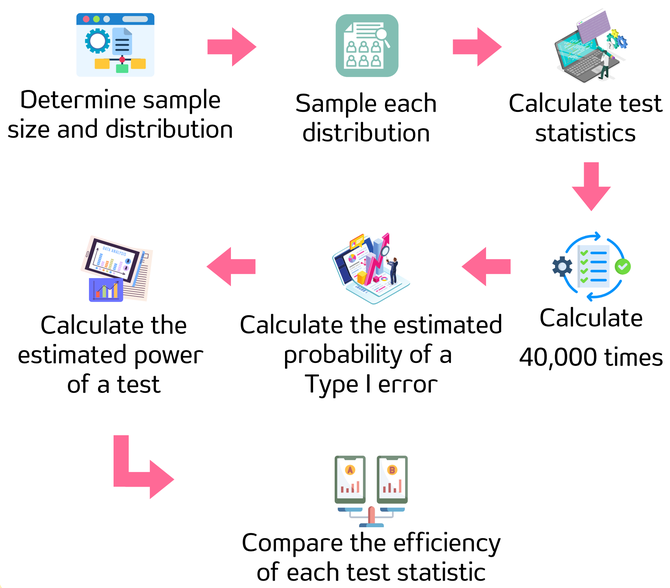
OBJECTIVE

Compare the efficiency of the Fligner-Killeen test, Ansari-Bradley test, and Moses test for testing homogeneity of variances between two populations.

SCOPE OF RESEARCH

1. Test statistics used to test the equality of variances of 2 populations
   - Fligner-Killeen test
   - Ansari-Bradley test
   - Moses test
2. The sample groups used in the study are the cases of equal sample sizes and unequal sample sizes, defined as follows
   - Case of equal sample sizes : (10,10), (20,20), (30,30)
   - Case of unequal sample sizes : (20,10), (30,15), (40,20)
3. Distribution of the data used for testing
   - Normal distribution
   - Beta distribution
   - Weibull distribution
4. Set the parameters at variance ratios of 1, 1.5, 2.25, and 3.375.
5. The significance level of the test is 0.05

METHODOLOGY



CONCLUSION

This study investigated the ability of the Fligner-Killeen, Ansari-Bradley, and Moses tests to control the estimated Type I error rate under Bradley's criterion. The results indicated that all three tests were able to control the Type I error rate across normal, beta, and Weibull distributions. These findings are consistent with Sukanya Yodnuan et al. (2024), who reported that Fligner-Killeen test effectively controls the Type I error rate in most Weibull distribution scenarios.

When the data follow a normal distribution, for both equal and unequal sample sizes, under the conditions where the sample sizes vary according to the variance ratio and where the sample sizes vary inversely with the variance ratio, Fligner-Killeen test provides the highest estimated test power. This result is consistent with the study of Adebayo and Ahmed (2024), which reported that when the data follow a normal distribution, the Fligner-Killeen test yields high estimated power across all sample sizes.

When the data follow a beta distribution, in the case of equal sample sizes, Ansari-Bradley test provides the highest estimated test power in almost all situations. For unequal sample sizes, when the sample sizes vary according to the variance ratio, Ansari-Bradley test yields the highest estimated test power in most situations. However, when the sample sizes vary inversely with the variance ratio, Fligner-Killeen test provides the highest estimated test power in almost all situations.

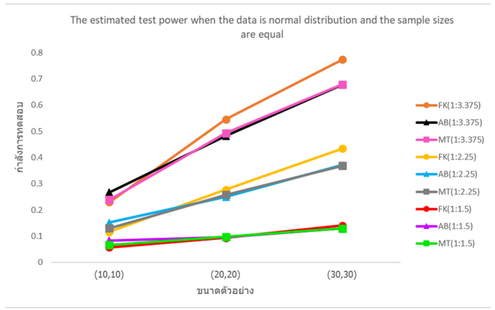
When the data follow a Weibull distribution, for both equal and unequal sample sizes, under the conditions where the sample sizes vary according to the variance ratio and where the sample sizes vary inversely with the variance ratio, Fligner-Killeen test provides the highest estimated test power in all situations.

RESULT

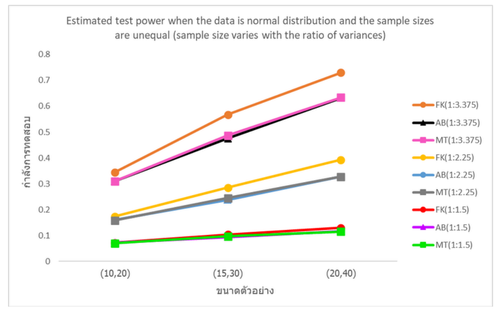
1. Comparing the ability to control the probability estimation of type I error rate

The study on the ability to control the probability estimation of type I error showed thatwhen the data were normal distribution, beta distribution, and Weibull distribution, Fligner-Killeen test, Ansari-Bradley test, and Moses test can control the estimation of the probability of type I error in all cases.

2. Comparing the estimated power of a test



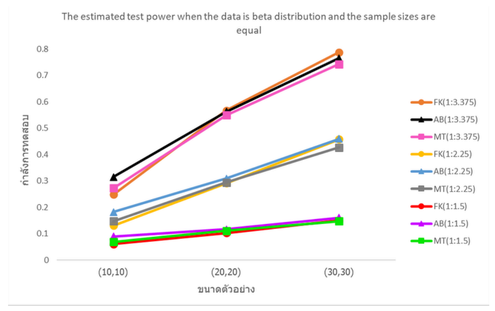
When the data are normal distribution and the sample sizes are equal, Fligner-Killeen test gives the highest power estimate in almost all cases.



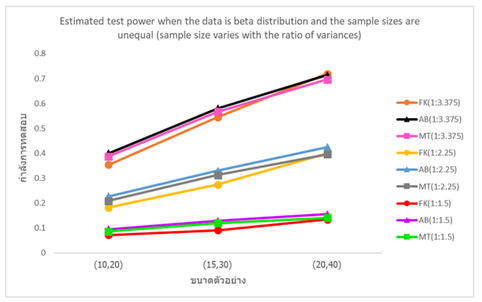
When the data is normal distribution and the sample sizes are unequal (sample size varies with the ratio of variance), Fligner-Killeen test yields the highest power estimate in almost all cases.



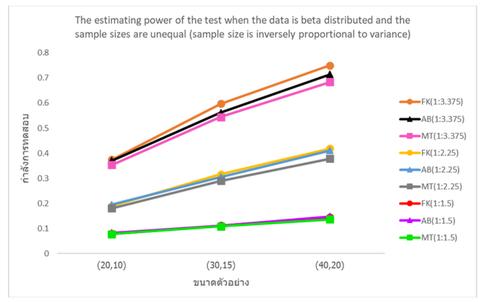
When the data are normal distribution and the sample sizes are unequal (sample sizes is inversely proportional to variance), Fligner-Killeen test yields the highest power estimate in almost all cases.



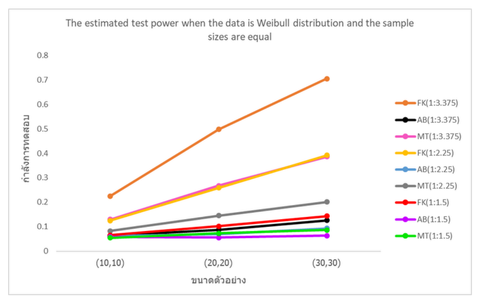
When the data have a beta distribution and the sample sizes are equal, Ansari-Bradley test yields the highest power estimate in almost all cases.



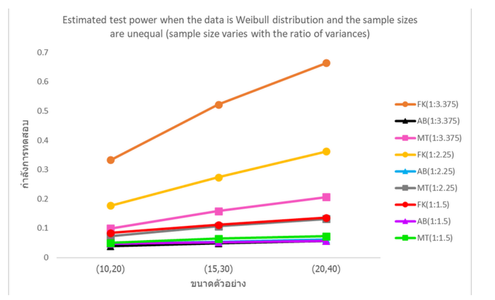
When the data is beta distribution and the sample sizes are not equal (sample size varies with the ratio of variance), Ansari-Bradley test gives the highest power estimate in almost all cases.



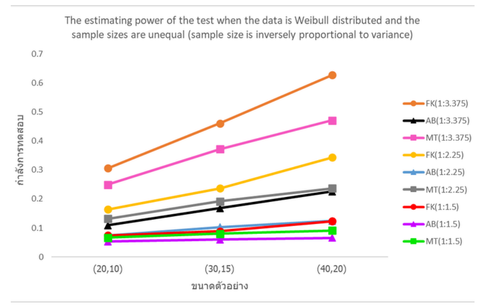
When the data is beta distributed and the sample sizes are unequal (sample sizes is inversely proportional to variance), Fligner-Killeen test gives the highest power estimate in almost all cases.e.



When the data is Weibull distribution and the sample sizes are equal, Fligner-Killeen test provides the highest power estimate in all cases.



When the data is Weibull distribution and the sample sizes are unequal (sample size varies with the ratio of variance), Fligner-Killeen test provides the highest power estimate in all cases.



When the data is Weibull distribution and the sample sizes are unequal (sample sizes is inversely proportional to variance), Fligner-Killeen test provides the highest power estimate in all cases.

Reference : Sukanya Yodnuan, Chularat Chumnuan, & Korkot Wijitsanguan Chetwan. (2023). An efficiency comparison of test statistics for testing homogeneity of variance under non-normal distributions. Journal of Science and Technology, Mahasarakham University, 42(2), 123-135.
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Yonar, A., Yonar, H., & Uçan, O. N. (2024). A comparative analysis for homogeneity of variance tests. Communications in Statistics - Simulation and Computation. Advance online publication. https://doi.org/10.1080/03610918.2024.2332328