Some Properties of Hyperbolic k-Narayana Octonions

FACULTY OF SCIENCE
CHIANG MAI UNIVERSITY

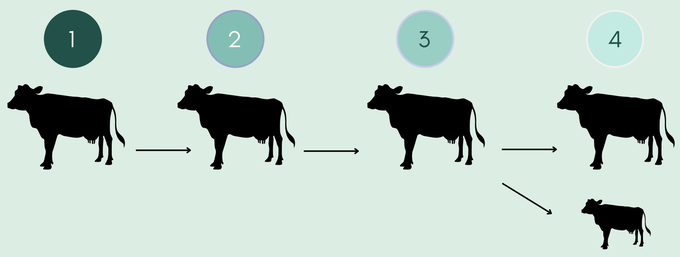
Abstract

This independent study focuses on the introduction and investigation of hyperbolic k-Narayana octonions. First, the definition and algebraic structure of hyperbolic k-Narayana octonions are established. Fundamental properties of these octonions are then examined. In particular, closed-form expressions in the form of Binet-type formulas are derived. Additionally, conjugation properties and summation formulas are obtained and analyzed. The results provide a foundational understanding of hyperbolic k-Narayana octonions and contribute to the broader study of generalized number sequences within higher-dimensional non-associative algebras.

Introduction

Hamilton invented quaternions in 1843. It can be used to show rotations in three dimensional space, and have applications in computer graphics, robotics, and aerospace engineering. Quaternions are also used in many areas of mathematics, such as differential geometry and number theory [1].
On the other hand, octonions are generalization of quaternions. In this work, we are interested in hyperbolic k-Narayana octonions, which are structures that generalize k-Narayana numbers. Chapter 2 will contain preliminaries while main results will be shown in Chapter 3. Finally, Chapter 4 offers conclusion.

Preliminaries



The Narayana numbers : In the 14th century,Tadepalli Venkata Narayana introduced them as follows

$$N_n = N_{n-1} + N_{n-3}, n \geq 3$$

with 0,1,1,1 initial condition. That is, the Narayana sequence is {0, 1, 1, 1, 2, 3, 4, 6, 9, 13,19, 28, 41, 60, ...}.

Objective

1. To study the definition and structure of hyperbolic k-Narayana octonions.
2. To prove some basic properties of hyperbolic k-Narayana octonions, such as the Binet's formulas,conjugate, and summation formulas.

Preliminaries

k-Narayana number : The k-Narayana numbers were introduce in 2015 by Ramírez and Sirvent

$$N_{k,n} = kN_{k,n-1} + N_{k,n-3}, n \geq 3$$

Result

k-Narayana octonion : For non-zero integers number k, and let $n \geq 0$ be an integer.The hyperbolic k-Narayana octonions $HON_{k,n}$ is defined by

$$\begin{aligned} HON_{k,n} = N_{k,n} + N_{k,n+1}i_1 + N_{k,n+2}i_2 + N_{k,n+3}i_3 \\ +N_{k,n+4}i_4 + N_{k,n+5}i_5 + N_{k,n+6}i_6 + N_{k,n+7}i_7, \end{aligned}$$

Some basic properties of hyperbolic k-Narayana octonions :

1. $HON_{k,n} + \overline{HON_{k,n}} = 2N_{k,n}.$
2. $HON_{k,n}\overline{HON_{k,n}} = 2HON_{k,n} - 2N_{k,n}.$
3. $HON_{k,n}\overline{HON_{k,n}} = \sum_{i=0}^{3} N_{k,n+i}^2 - \sum_{i=4}^{7} N_{k,n+i}^2$

The Binet's formulas of hyperbolic k-Narayana octonion

$$HON_{k,n} = \frac{\alpha_k^{n+1}\hat{\alpha}_k}{(\alpha_k - \beta_k)(\alpha_k - \gamma_k)} + \frac{\beta_k^{n+1}\hat{\beta}_k}{(\beta_k - \alpha_k)(\beta_k - \gamma_k)} + \frac{\gamma_k^{n+1}\hat{\gamma}_k}{(\gamma_k - \alpha_k)(\gamma_k - \beta_k)}$$

The summation for the hyperbolic k-Narayana octonion

$$\begin{aligned} \sum_{m=0}^{n} H\mathfrak{R}_{k,m} &= \frac{N_{k,n} + N_{k,n+1} + N_{k,n+2} - kN_{k,n+1} + 1 + \mathbf{i}_1 + \mathbf{i}_2 + \mathbf{i}_3 + \mathbf{i}_4 + \mathbf{i}_5 + \mathbf{i}_6 + \mathbf{i}_7}{k} \\ &\quad -\mathbf{i}_2 - (k+1)\mathbf{i}_3 - (k^2+k+1)\mathbf{i}_4 - (k^3+k^2+k+2)\mathbf{i}_5 \\ &\quad -(k^4+k^3+k^2+3k+2)\mathbf{i}_6 - (k^5+k^4+k^3+4k^2+3k+2)\mathbf{i}_7 \end{aligned}$$

References

- [1] Godase, A. D. (2024). Some new properties of hyperbolic k-Fibonacci and k-Lucas octonions.
- [2] Engin Ö., Bahar K., & James F, P.(2021). k-Narayana sequence self-similarity.
- [3] Horadam, A. (1963). Complex Fibonacci numbers and Fibonacci quaternions. The American Mathematical Monthly, 70(3), 289–291.
- [4] Macfarlane, A. D. (1900). Hyperbolic quaternions. Proceedings of the Royal Society of Edinburgh, 23, 169–180.
- [5] Tanupat P.,& Narawadee P.(2024). Some Properties of Hyperbolic k-Narayana Quaternions
- [6] Carmody, K. (1988). Circular and hyperbolic quaternions, octonions, and sedenions.Applied Mathematics and Computation, 28(1), 47-72.

Bunnawit Pichaiwong 650510581
Advisor : Assoc. Prof. Dr.Narawadee Phudolsitthiphat
Department of Mathematics, Chiang Mai University
Email : eqbunnawit@gmail.com