AN EDGE-INDEPENDENT SET OF COMPLETE TRIPARTITE GRAPHS $K_{n,n,n}$

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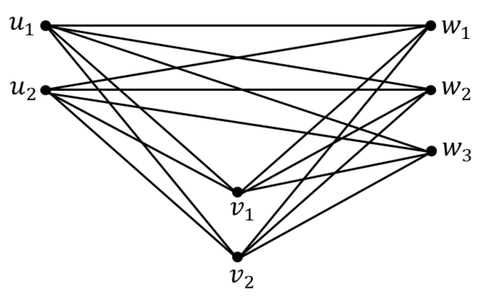
ABSTRACT

In this independent study we explore the chromatic number, chromatic index and total chromatic number of simple graphs through the analysis of independent sets. The research primarily focuses on determining the chromatic index of complete tripartite graphs $K_{m,n,k}$. The findings demonstrate that for the complete tripartite graph $K_{n,n,n}$, the chromatic index is $2n$ when n is an even integer. Conversely, when n is an odd integer, the chromatic index is $2n+1$. Furthermore, the study investigates the properties of chromatic numbers in isomorphic graphs and almost complete tripartite graphs to establish a theoretical foundation for proofs. These results contribute to extending the research toward establishing conjectures for the chromatic index of complete tripartite graphs $K_{m,n,k}$ in the general case.

PRELIMINARIES

Complete Tripartite Graph

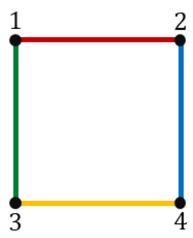
A graph whose vertices can be partitioned into three disjoint sets such that no edges exist within each set, but every vertex is connected to all vertices in the other sets. The graph is denoted by $K_{m,n,k}$.



A complete tripartite graph $K_{2,2,3}$

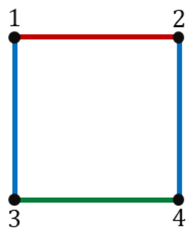
Edge-coloring

Edge-coloring of a graph is a coloring of the edges such that adjacent edges have distinct colors.



Independent set

Let G be a simple graph. A subset I of the edge set $E(G)$ is said to be edge-independent if all edge in the set are not adjacent to the others.

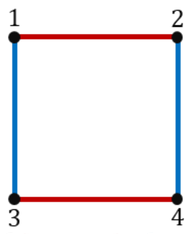


Example of edge-indenpendent sets: $\{12\},\{34\},\{13,24\}$

The chromatic index

The chromatic index $\chi'(G)$ of a graph G is the least number of colors needed to color the edges such as, edge-coloring of G.

We will construct the smallest partition of an independent set in order to determine the chromatic index.



$P = \{\{12,34\},\{13,24\}\}$

INTRODUTION

In the previous work, we explored three typs of colorings on complete graphs and complete bipartite graphs, namely vertex-coloring, edge-coloring and total-coloring.The concept of coloring is related to constructing independent sets, which serves as the motivation for our work.Therefore, in this study, we investigate these three typs of colorings on complete tripartite graphs through the perspective of independent sets.

METHODOLOGY

- Review definitions and fundamental concepts of graph coloring.
- Apply known results on complete bipartite graphs and almost complete tripartite graphs.
- Analyze independent sets and their partitions.
- Determine the chromatic index of complete tripartite graphs.

RESULTS

1.The chromatic index of complete tripartite graph

$\chi'(K_{m,n,k}) = k + max\{m,n\}$ where $k \geqslant m+n.$

2.The chromatic index of complete tripartite graph

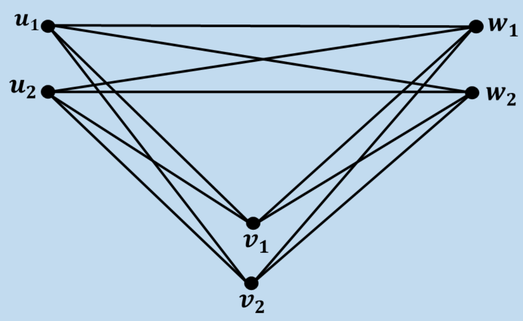
$\chi'(K_{n,n,n}) = 2n$ where n is even.

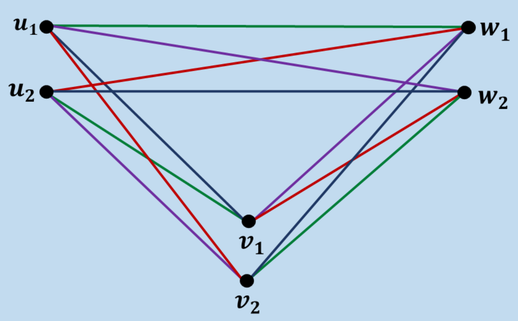
3.The chromatic index of complete tripartite graph

$\chi'(K_{n,n,n}) = 2n+1$ where n is odd.

CONCLUSION

This study investigates the chromatic index of complete tripartite graphs by considering edge independent sets. It is found that the differences in the values of m,n,k affect the edge chromatic index. Therefore, this research considers three cases based on the relationships among m,n,k in order to analyze and determine the chromatic index of complete tripartite graphs.





REFERENCE

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[2] V.G. Vizing, On the estimate of the chromatic class of a p-graph, Diskret. Analiz 3 (1964) 25-30.
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