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An ODE Model for IBNR Estimation Using Cumulative Paid and Outstanding Case Reserves

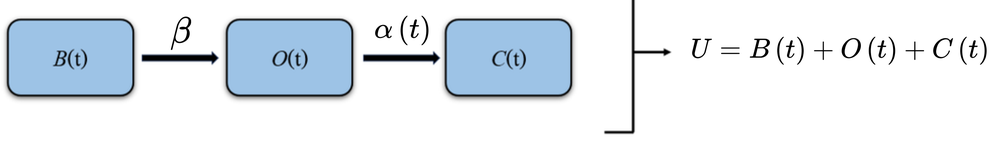
Abstract Estimating incurred but not reported (IBNR) claims is an important task in loss reserving. Although the Chain-Ladder method is widely used in practice, it does not clearly describe how claims develop over time. This study proposes an ordinary differential equation (ODE) model to represent the relationships among paid losses, case reserves for reported claims, and IBNR by using claims data from 2019–2024 and estimating the parameters α_0, λ and β by using numerical approximation and linear regression. The results show that the proposed model provides IBNR estimates that are structurally consistent and reasonable for insurance applications, with almost no negative values, while the Chain-Ladder method produces negative estimates in several cohorts. Backtesting shows that both methods have similar predictive accuracy and the proposed model offers a clearer structural framework and meaningful interpretation of parameters for insurance practice.

Introduction

Non-life insurance companies need to keep enough claim reserves to meet their obligations from insurance contracts and stay financially stable. However, the traditional chain-ladder methods, which use past claim data, cannot fully capture how claims change over time or the relationship between paid claims and outstanding reserves. This study aims to develop an ordinary differential equation (ODE) model to describe the relationships among paid claims, case reserves, and IBNR, providing a clear mathematical framework to improve claim estimation and support better reserving decisions.

Methodology

Model



$$\frac{dC}{dt} = \alpha(t)O(t)$$

$$\frac{dO}{dt} = \beta[U - O(t) - C(t)] - \alpha(t)O(t)$$

- U = Ultimate loss
- $B(t)$ = IBNR (Incurred But Not Reported)
- $O(t)$ = Outstanding Case Reserve
- $C(t)$ = Cumulative Paid
- β = Claim reporting rate
- $\alpha(t)$ = Claim payment rate

$$\alpha(t) = \alpha_0 e^{-\lambda t}$$

where
- α_0 = Initial claim payment rate
- λ = Claim closure decay rate

Research Process

Data Preparation
(Motor insurance data Q1 2019 - Q4 2024)
Ref: https://www.oic.or.th/

Parameter estimation
(Find α_0 , λ , β , U
by Numerical Approximation & Linear Regression)
via MS Excel

IBNR Estimation Models
via MS Excel

Standard Chain-Ladder Method

ODE Model (Constant $\alpha(t)$)

$$C(t) = Ae^{-\alpha t} + Be^{-\beta t} + U$$
$$O(t) = -Ae^{-\alpha t} - \frac{\beta}{\alpha}Be^{-\beta t}$$

ODE Model (Euler Method)

$$C_{n+1} = C_n + \alpha_n O_n$$
$$O_{n+1} = O_n + \beta[U - O_n - C_n] - \alpha_n O_n$$

constraints $C_{n+1} \geqslant C_n$

$O_n \geqslant 0$

Model Evaluation
(Backtesting & RRMSE)

Results

IBNR Estimation

Chain-ladder	Euler method	Constant
-	-	-
-	-	-
-	71,531.71	-
-	-	-
-4,409.59	-1,734.21	-2,897.41
-11,746.69	593,686.14	-13,881.75
-5,824.35	-4,706.74	-13,370.93
-517.6	-3,242.82	-11,997.38
-8,245.72	-3,866.75	-19,029.79
-11,117.13	32,513.24	-21,570.60
-7,777.16	1,928,330.99	-17,844.97
-36,079.30	329,443.64	-49,662.61
-25,636.08	1,641,147.44	-42,767.92
2,072.32	691,531.92	-24,930.84
3,210.36	4,035.53	-37,035.13
-131,809.94	29,326.93	-182,672.42
-407,663.37	103,357.54	-483,566.61
-457,047.89	641,172.80	-579,830.45
-401,301.58	1,530,609.59	-579,294.39
-370,150.89	1,292,093.47	-683,528.00
-188,540.44	2,762,862.37	-665,168.09
208,779.08	4,174,342.00	-570,881.51
1,125,273.72	5,035,750.41	-664,733.74
871,278.29	6,178,981.15	-2,702,523.48
142,746.05	27,027,166.33	-7,367,178.03

Chain-ladder
- IBNR = 142,746.05 baht
- Negative values occur in several cohorts, leading to mutual offsetting effects.
- IBNR is unrealistically low.

Euler method
- IBNR = 27,027,166.33 baht
- Negative values are rarely observed due to the imposed constraints.
- IBNR is appropriate or overestimated.

Constant
- IBNR = -7,367,178.03 baht
- Many negative values occurred due to excessive claim payments.
- IBNR is inappropriate.

Negative IBNR values indicate that the paid claims exceed the estimated ultimate claims, which reduces the reliability of the estimates.

Model performance

Method	RRMSE	Business Interpretability
Chain-ladder	**0.85%**	**Black-Box**
Euler method	**1.10%**	**White-Box**

The ODE model offers comparable accuracy with the added benefit of white-box interpretability.

Business Interpretation

Parameter	Evaluation Criteria	Estimated Value	Business Conclusion
α_0	$\alpha_0 \geqslant 1.0$ (Aggressive Payout)	$\alpha_0 = 1.724$	Claims are paid quickly in the early development quarters.
	$0.5 \leqslant \alpha_0 < 1.0$ (Moderate Payout)		
	$\alpha_0 < 0.5$ (Slow Payout)		
λ	$\lambda \geqslant 0.05$ (Steep Decay)		Claims are closed at a moderate speed.
	$0.03 \leqslant \lambda < 0.05$ (Gradual Decay)	$\lambda = 0.045$	
	$\lambda < 0.03$ (Constat Decay)		
β	$\beta \geqslant 0.34$ (Fast Reporting)		Claims are reported at a moderate speed.
	$0.17 \leqslant \beta < 0.34$ (Medium Reporting)	$\beta = 0.227$	
	$\beta < 0.17$ (Slow Reporting)		

Note: The evaluation criteria and estimated timeframes (quarters) are derived using the Half-life approach.

Conclusion

The ODE model proposed provides a clear and transparent approach for estimating IBNR claims in a manner that aligns with the actual progression of claims through various stages. The key parameters (α_0 , λ , β) are fitted to translate the claims process into business terms, thereby “spotlighting” how claims payments progress and how reporting works. It has similar predictive capability to the standard method but nicely sidesteps two common issues with the Chain-Ladder method, nonsensical negative numbers and under-reserving risks. The final result offers insurers a clear and reliable tool to determine claim reserves in business terms.

References

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