Chiang Mai J. Sci. 2017; 44(4) : 1735-1743
http://epg.science.cmu.ac.th/ejournal/
Contributed Paper

Numerical Solution of Multi-Asset Option Pricing Problems Using an Improved RBF-DQ Method

Leila Khodayari and Mojtaba Ranjbar
Department of Mathematics, Azarbaijan Shahid Madani University, Tabriz, Iran.
*Author for correspondence; e-mail: leila.khodayari@azaruniv.edu and m_ranjbar@azaruniv.edu

Received: 4 January 2015
Accepted: 3 February 2016

ABSTRACT

In this paper, a modification of the original global radial basis functions-based differential quadrature (RBF-DQ) method is set forth and analyzed. The improved RBF-DQ method is applicable to the numerical approximation of solutions of a wide range of partial differential equations (PDEs) with mixed derivative terms. However, it appears to be considerably faster than the original method. In support of this contention, the multidimensional Black-Scholes (BS) equation in the Geometric Brownian Motion (GBM) framework has been solved numerically by using the proposed method and compared with results obtained via the original RBF-DQ method. For accuracy achieved versus work expended, the improved method performs better.

Keywords: radial basis functions, multi-dimensional Black-Scholes equation, differential quadrature, european option

1. INTRODUCTION

The original differential quadrature (DQ) method for the numerical approximation of PDEs was developed by Bellman et al. in [1, 2]. The basic idea of the DQ method is that any derivative at a mesh point, can be approximated by a weighted linear sum of the function values along a mesh line. The key step in the DQ method is determination of weighting coefficients to determine a discrete approximation of any order derivative. As has been shown in [3], RBFs which are mesh-free and insensitive to dimension present a good choice for use in the DQ approximation. Exploiting the advantages of RBFs, Shu et al. proposed an alternative set of mesh-less methods which they termed RBF-DQ methods. In their scheme, the RBFs are taken as the test functions in the DQ approximation to compute the weighting coefficients. The method not only inherits the high accuracy and efficient computation of the DQ method, but also possesses desirable features of RBFs such as being mesh-free and easily extensible to higher dimensions. In this study we improve the RBF-DQ method. The improved RBF-DQ method can be used to approximate the solutions of a wide range of PDEs with mixed derivative terms. Moreover, the improved method is much faster than the original one as we will presently indicate by example.

Options are an important and widely used class of financial derivatives. A major concern in financial markets is determining the value of an option. An option contract is an agreement between a buying party (the holder) and a selling party (the underwriter). The holder of the option contract has no obligation to exercise his option contract, but if the holder chooses to do so, the underwriter is obliged to execute the contract. Here, we will be particularly concerned with options where the payoff depends on multiple underlying assets. The pricing of such option can be modeled by a higher dimensional generalization of the original equation proposed by Black and Scholes [4], as extended by Merton [5], for evaluating European call options without dividends [4].

By assuming that the asset price is risk-neutral, Black and Scholes showed that the value of a single asset European call option satisfies a lognormal diffusion type PDE, now commonly referred to as the BS equation. If we let $V\ t,s$ represent the option value at time t with stock prices $S = (s_1, s_2, \cdots, s_d)$, then we get the following linear parabolic PDE

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sum_{i,j=1}^{d} \rho_{ij}\sigma_i\sigma_j s_i s_j \frac{\partial^2 V}{\partial s_j \partial s_j} + \sum_{i=1}^{d} r s_i \frac{\partial V}{\partial s_i} - rV = 0, \tag{1}$$

which is known as the BS equation for multi-asset option problems. Respectively s_i, σ_i, ρ_{ij} and r are, i-th asset price, volatility of the i-th asset price, correlation between the prices of i-th and j-th assets, and the risk free interest rate. It is very hard to get an analytic form solution for most financial derivatives due to the complexity of the financial product itself and the complexity of the financial market; however, various numerical techniques can be applied to price multi-variate derivatives. Numerical methods using higher-dimensional generalizations of lattice binomial methods can be used cf. [6], for European options based on three underlying assets. Stable higher-order methods for the BS equation have been introduced by Voss et al. [7] and Khaliq et al. [8]. Moreover, mesh-free methods based on RBFs may also reduce computational cost significantly, [9]. Jo and Kim in [10] combined the operator splitting method with a parallel computation technique for solving the multi-dimensional BS equations. Nielsen applied penalty methods for the numerical solution of American multi-asset option problems [11]. Martin in 2014 stabilized proposed explicit Runge-Kutta methods for multi-asset American options [12].

The organization of this paper is as follows: In Section 2, the improved RBF-DQ method for approximation of partial derivatives is described. Section 3 is devoted to adapting this method to approximate multi-dimensional option pricing. In Section 4 we discuss the stability of the method in the context of the multi-asset option pricing problems, while Section 5 provides telling numerical examples comparing the performance of the new method with the original RBF-DQ method. The paper concludes in Section 6 with a brief retrospective.

2. RBF-BASED DQ METHOD

In this section, we will show the RBF-DQ method and the recommended modification to it in detail.

2.1 Radial Basis Functions

Interpolation theory of RBFs has been the subject of intensive research in recent decades and nowadays RBFs play an increasingly important role in the field of reconstructing functions from multivariate scattered data. In general terms, the interpolation theory of RBFs can be described as follows: If an unknown function $f(X)$ is only known at a finite set of centres $X_i, i = 1, \cdots, N$, then f can be approximated as a linear combination of N, RBFs

$$f(X) \cong \sum_{j=1}^{N} \lambda_j \varphi(X, X_j) + \boldsymbol{P}(X), \qquad X \in \Omega \subset \mathbb{R}^d, \tag{2}$$

where N is the number of data points, $X = (x_1, x_2, \cdots, x_d)$, d is the dimension of the problem, the λ's are coefficients to be determined and φ is the RBF. Eq. (2) may also be written without the polynomial term $\boldsymbol{P}$. Among the tested RBFs, multi-quadrics (MQs) yield the most accurate results. MQ-RBFs can be written as

$$\varphi(X, X_j) = \varphi(r_j) = \sqrt{r_j{}^2 + c_j{}^2}\,, \tag{3}$$

where $r_j = |X - X_j|$ is the usual Euclidian distance and c_j is a shape parameter. If we let $\boldsymbol{P}_q^d$ denote the space of d-variate polynomials of order not exceeding q and choose the polynomials $P_1, P_2, \cdots, P_m$ a basis of $\boldsymbol{P}_q^d$ in $\mathbb{R}^{\boldsymbol{d}}$, then the polynomial $\boldsymbol{P}(X)$ in Eq. (2), is usually written in the following form:

$$\boldsymbol{P}(X) = \sum_{i=1}^{m} \xi_i P_i(X), \tag{4}$$

where $= \frac{(q-1+d)!}{d!(q-1)!}$. To determine the coefficients $(\lambda_1, \lambda_2, \cdots, \lambda_N)$ and $(\xi_1, \xi_2, \cdots, \xi_m)$, an extra m equations are required in addition to the N equations resulting from collocating Eq. (2) at the N points. This is ensured by the m conditions for Eq. (2),

$$\sum_{j=1}^{N} \lambda_j P_i(X_j) = 0, i = 1, \cdots, m. \tag{5}$$

2.2 DQ Method for Partial Derivative Approximation in Higher Dimensions

In this section, we explain the method to approximate the spatial derivative in two-dimensions. It is noted that the basic idea of the DQ method is that any derivative can be approximated by a linear weighted sum of the function values at some mesh points. We retain this idea but relax the condition of choosing function values along a mesh line. In other words, for a two-dimensional problem, any spatial derivative is approximated by a linear weighted sum the function values on the whole two-dimensional domain. In this approximation, a mesh point in the two-dimensional domain is represented by one index k, while in the conventional DQ approximation, the mesh point is represented by two indices i, j. If the mesh is structured, it is easy to establish the relationship between i, j, and k. For example k can be written as $k = (i-1)N_2 + j, i = 1, \cdots, N_1;\ j = 1, \cdots, N_2$. As i varies from 1 to N_1 and j varies from 1 to N_2, k varies over 1 to $N = N_1 \times N_2$. *The* DQ approximation for the n-th-order derivative with respect to x, and the m-th-order derivative with respect to y at (x_k, y_k) can then be written as

$$\frac{\partial^n f}{\partial x^n}(x_k, y_k) = \sum_{z=1}^{N} w_{k,z}^{(n)_x} f(x_z, y_z), \tag{6}$$

$$\frac{\partial^m f}{\partial y^m}(x_k, y_k) = \sum_{z=1}^{N} w_{k,z}^{(m)_y} f(x_z, y_z). \tag{7}$$

Shu et al. employed this technique and made a novel and effective algorithm for the use of RBFs to solve the PDEs [3]. Instead of using polynomials for determining coefficients, they applied RBFs as test functions. In order to apply the original DQ method to approximate the

PDE with mixed derivative terms, they should be removed by making some change of coordinates. However, we can improve the DQ method by using Eq. (6) and Eq. (7) so that we can easily approximate a PDE with mixed derivative terms as follows:

$$\begin{aligned}\frac{\partial^{n+m} f}{\partial x^n \partial y^m}(x_k, y_k) &= \frac{\partial}{\partial x}\left(\frac{\partial^{n+m-1} f}{\partial x^{n-1} \partial y^m}(x_k, y_k)\right) \\ &= \sum_{z_1=1}^{N} w_{k,z_1}^{(1)_x} \frac{\partial^{n+m-1} f}{\partial x^{n-1} \partial y^m}(x_{z_1}, y_{z_1}) \\ &\vdots \\ &= \sum_{z_1,\cdots,z_{n+m}=1}^{N} \underbrace{w_{k,z_1}^{(1)_x} w_{z_1,z_2}^{(1)_x} \cdots w_{z_{n-1},z_n}^{(1)_x}}_{n} \underbrace{w_{z_n,z_{n+1}}^{(1)_y} \cdots w_{z_{n+m-1},z_{n+m}}^{(1)_y}}_{m} f(x_{z_{n+m}}, y_{z_{n+m}}). \qquad (8)\end{aligned}$$

By rewriting the right hand side of Eq. (6) and Eq. (7) as

$$\sum_{z_1,\cdots,z_n=1}^{N} \underbrace{w_{k,z_1}^{(1)_x} w_{z_1,z_2}^{(1)_x} \cdots w_{z_{n-1},z_n}^{(1)_x}}_{n} f(x_{z_n}, y_{z_n}), \quad \sum_{z_1,\cdots,z_m=1}^{N} \underbrace{w_{k,z_1}^{(1)_y} w_{z_1,z_2}^{(1)_y} \cdots w_{z_{m-1},z_m}^{(1)_y}}_{m} f\left(x_{z_m}, y_{z_m}\right),$$

we can also decrease the computation time significantly. In the following subsection, we will show that the weighting coefficients $w_{i,j}^{(1)_x}$ and $w_{i,j}^{(1)_y}$ can be determined by the aforementioned function approximation of RBFs and the analysis of a linear vector space.

2.3 RBF-DQ Approximation

In this subsection, we will use the MQ-RBFs as the test functions to determine the weighting coefficients in the DQ approximation of derivatives for a two-dimensional problem. Suppose that the solution of a PDE is continuous, which can be approximated by MQ-RBFs in Eq. (3) and only a constant is included in the polynomial term $\boldsymbol{P}(x, y)$. So, for a two-dimensional case, the function approximation can be written as

$$f(X) \cong \sum_{j=1}^{N} \lambda_j \varphi(|(x, y) - (x_j, y_j)|) + \mu, \qquad (9)$$

Eq. (9) has $N+1$ unknown coefficients and can only be applied at N nodes. So, we need an additional equation to close the system. To make the problem be well-posed, this additional equation can be made by Eq. (5) in which the sum of the expansion coefficients to be zero. As a result, we have

$$\sum_{j=1}^{N} \lambda_j = 0 \Rightarrow \lambda_i = -\sum_{j=1, j\neq i}^{N} \lambda_j, \qquad (10)$$

substituting Eq. (10) into Eq. (9) gives

$$f(X) \cong \sum_{j=1, j\neq i}^{N} \lambda_j \left(\varphi(|(x, y) - (x_j, y_j)|) - \varphi(|(x, y) - (x_i, y_i)|)\right) + \mu, \qquad (11)$$

the number of unknowns in Eq. (9) is reduced to N. As no confusion rises, μ can be replaced by λ_i and Eq. (11) can be written as

$$f(x, y) \cong \sum_{j=1, j\neq i}^{N} \lambda_j \left(\varphi(|(x, y) - (x_j, y_j)|) - \varphi(|(x, y) - (x_i, y_i)|)\right) + \lambda_i. \qquad (12)$$

By setting $g_j(x, y) = \varphi(|(x, y) - (x_j, y_j)|) - \varphi(|(x, y) - (x_i, y_i)|), j = 1, \cdots, i-1, i+1, \cdots, N$, Eq. (12) can be further written as

$$f(x,y) \cong \sum_{j=1, j\neq i}^{N} \lambda_j g_j(x,y) + \lambda_i, \tag{13}$$

the form of Eq. (13) constructs an N-dimensional linear vector space V^N. A set of base functions in V^N can be taken as $q_j = g_j(x,y) = \varphi(|(x,y) - (x_j, y_j)|) - \varphi(|(x,y) - (x_i, y_i)|), j = 1, \cdots, i-1, i+1, \cdots, N,\ q_i = 1$. From the concept of linear independence, the bases of a vector space can be considered as linearly independent subset that spans the entire space. From the property of a linear vector space, if all the base functions satisfy the linear Eq. (6) and Eq. (7), so does any function in the space V^N represented by Eq. (13). There is an interesting feature. From Eq. (13), while all the base functions are given, the function $f(x,y)$ is still unknown since the coefficients λ_i are unknown. However, when all the base functions satisfy Eq. (6) and Eq. (7), we can guarantee that $f(x,y)$ also satisfies Eq. (6) and Eq. (7). In other words, we can guarantee that the solution of a PDE approximated by the RBF satisfies Eq. (6) and Eq. (7). Thus, when the weighting coefficients of DQ approximation are determined by all the base functions, they can be used to discretize the derivatives in a PDE. That is the essence of the RBF-DQ method. Using the idea of DQ method, the weighting coefficients of the first order partial derivatives can be determined by substituting the entire base functions $q_1, \cdots, q_N$ into Eq. (6), as

$$\frac{\partial q_i(x_i, y_i)}{\partial x} = \sum_{k=1}^{N} w_{i,k}^{(n)_x} = 0, \tag{14}$$

$$\frac{\partial q_j(x_i, y_i)}{\partial x} = \sum_{k=1}^{N} w_{i,k}^{(n)_x} q_j(x_k, y_k), j = 1, \cdots, N, j \neq i, \tag{15}$$

For the given i, equation system (14) and (15) has N unknowns with N equations. The matrix form for the weighting coefficients can be written as

$$\boldsymbol{q}_i^{(n)_x} = [\boldsymbol{Q}]\boldsymbol{W}_i^{(n)_x}, \tag{16}$$

where

$$\boldsymbol{q}_i^{(n)_x} = [0, \frac{\partial q_1(x_i, y_i)}{\partial x}, \cdots, \frac{\partial q_{i-1}(x_i, y_i)}{\partial x}, \frac{\partial q_{i+1}(x_i, y_i)}{\partial x} \cdots, \frac{\partial q_N(x_i, y_i)}{\partial x}]^T, \boldsymbol{W}_i^{(n)_x} = [W_{i,1}^{(n)_x}, \cdots, W_{i,N}^{(n)_x}]^T,$$

and

$$[\boldsymbol{Q}] = \begin{bmatrix} 1 & \cdots & 1 \\ q_1(x_1, y_1) & \cdots & q_1(x_N, y_N) \\ \vdots & \ddots & \vdots \\ q_{i-1}(x_1, y_1) & \cdots & q_{i-1}(x_N, y_N) \\ q_{i+1}(x_1, y_1) & \cdots & q_{i+1}(x_1, y_1) \\ \vdots & \ddots & \vdots \\ q_N(x_1, y_1) & \cdots & q_N(x_N, y_N) \end{bmatrix}.$$

Clearly, there exists a unique solution only if the collocation matrix $[\boldsymbol{Q}]$ is non-singular. The non-singularity of the collocation matrix $[\boldsymbol{Q}]$ depends on the properties of used RBFs. Micchelli [13] proved that matrix $[\mathbf{Q}]$ is conditionally positive definite for MQ-RBFs. This fact cannot guarantee the non-singularity of matrix $[\mathbf{Q}]$. Hon [14] showed that cases of singularity are quite rare and not serious objection to a valuable numerical technique. Therefore, by using Eq. (16) with $n = 1$, the coefficient vector $\boldsymbol{W}_{\mathrm{i}}^{(1)_x}$ can be obtained as $\boldsymbol{W}_i^{(1)_x} = [\boldsymbol{Q}]^{-1}\boldsymbol{q}_i^{(1)_x}$. Then, the coefficient vector $\boldsymbol{W}_i^{(1)_x}$ can be used to approximate the first-order derivative in the x direction for any unknown smooth function at node i in Eq. (8). In a similar manner, all the base functions are substituted into Eq. (7) to approximate the first-order derivative in the y direction for any unknown smooth function at node i.

3. AN IMPRCVED GLOBAL RBF-DQ METHOD FOR MULTI-DIMENSIONAL BS EQUATION

In this section, to descritize the PDE (1) arising from GBM model: Let $U(t,X) = V(t,S)$,$x_i = log(s_i)$, $X = (x_1, \cdots, x_d)$ and payoff $g(S) = h(X)$. Then the PDE is rewritten as

$$\frac{\partial U}{\partial t} + \frac{1}{2}\sum_{i,j=1}^{d} \rho_{ij}\sigma_i\sigma_j \frac{\partial^2 U}{\partial x_i \partial x_j} + \sum_{i=1}^{d}\left(r - \frac{1}{2}\sigma_i^2\right)\frac{\partial U}{\partial x_i} - rU(t,X) = 0, \tag{17}$$

or $-\frac{\partial \mathrm{U}}{\partial \mathrm{t}} = \mathfrak{D}\mathrm{U}$ where $\mathfrak{D}$ represent the differential operator. The domain is discretized by taking N knots according to the method used in previous section and we would like to discrete the Eq. (17) with respect to time. We introduce a weight θ and apply the θ implicit scheme to the problem as

$$-\frac{\partial U}{\partial t}|_{(t_n, X_k)} \cong \frac{\widetilde{U}_k^n - \widetilde{U}_k^{n-1}}{\Delta t} = (1-\theta)\mathfrak{D}\widetilde{U}|_{(t_n,X_k)} + \theta\mathfrak{D}\widetilde{U}|_{(t_{n-1},X_k)}, \tag{18}$$

denote $\widetilde{U}|_{(t_n,X_k)} = \widetilde{U}|_{(t_n,x_1^k,\cdots,x_d^k)} = \widetilde{U}_k^n$, where represents an approximation for the function value of U at knot k and time t_n. By applying Eq. (8), Eq. (17) can be rewritten in a discrete form as follows:

$$\begin{aligned}(1+r(1-\theta)\Delta t)\widetilde{U}_k^n - (1-\theta)\Delta t\left(\sum_{i=1}^{d}\sum_{z=1}^{N}\left(r-\frac{1}{2}\sigma_i^2\right) w_{k,z}^{(1)x_i}\,\widetilde{U}_z^n + \sum_{i,j=1}^{d}\sum_{o,z=1}^{N}\rho_{ij}\sigma_i\sigma_j w_{k,o}^{(1)x_i} w_{o,z}^{(1)x_j}\widetilde{U}_z^n\right) \\ = (1-r\theta\Delta t)\widetilde{U}_k^{n-1} + \theta\Delta t\left(\sum_{i=1}^{d}\sum_{z=1}^{N}\left(r-\frac{1}{2}\sigma_i^2\right) w_{k,z}^{(1)x_i}\,\widetilde{U}_z^{n-1} + \sum_{i,j=1}^{d}\sum_{o,z=1}^{N}\rho_{ij}\sigma_i\sigma_j w_{k,o}^{(1)x_i} w_{o,z}^{(1)x_j}\widetilde{U}_z^{n-1}\right),\end{aligned} \tag{19}$$

where $w_{ks}^{(1)x_i}$ represents the computed weighting coefficients in the DQ approximation for the first order derivatives in the x_i direction in X_k.

4. STABILITY OF THE METHOD

In this subsection, we study the stability of the implicit finite difference method described above. Let us assume that $\boldsymbol{U}$ be exact and $\widetilde{\boldsymbol{U}}$ is the numerical solution of equation (17). Note that the discrete equations Eq. (19) can be rewritten as the following form

$$\widetilde{\boldsymbol{U}}_n = [\boldsymbol{I} + (1-\theta)\Delta t\boldsymbol{D}]^{-1}[\boldsymbol{I} - \theta\Delta t\boldsymbol{D}]\widetilde{\boldsymbol{U}}_{n-1} = \boldsymbol{E}\widetilde{\boldsymbol{U}}_{n-1}, \tag{20}$$

Where $\boldsymbol{D} = r - \sum_{i,j=1}^{d}\rho_{ij}\sigma_i\sigma_j\left([\boldsymbol{Q}]^{-1}[\boldsymbol{Q}]_{x_i}\right)^T\left([\boldsymbol{Q}]^{-1}[\boldsymbol{Q}]_{x_j}\right)^T - \sum_{i=1}^{d}\left(1-\frac{1}{2}{\sigma_i}^2\right)\left([\boldsymbol{Q}]^{-1}[\boldsymbol{Q}]_{x_i}\right)^T$, is $N\times N$ matrix determined by Eq. (19) in grids points and $[\boldsymbol{Q}]_{x_i} = [\boldsymbol{q}_1^{(1)x_i}, \boldsymbol{q}_2^{(1)x_i}, \cdots, \boldsymbol{q}_N^{(1)x_i}]$. The scheme (20) for initial value problem is stable if and only if there exists a positive constant C independent of the mesh spacing and initial data such that

$$\|\widetilde{\boldsymbol{U}}_n\| \le C\|\widetilde{\boldsymbol{U}}_0\|, n\to\infty, \Delta t, \Delta X \to 0.$$

If $\boldsymbol{E}$ depended on n we would get a product of the operators at each time level. Taking a norm

$$\|\widetilde{\boldsymbol{U}}_n\| = \|\boldsymbol{E}^n\widetilde{\boldsymbol{U}}_0\| \le \|\boldsymbol{E}^n\|\|\widetilde{\boldsymbol{U}}_0\|.$$

Therefore, the numerical scheme is stable if and only if there exists positive constant C such that

$$\|\boldsymbol{E}^n\| \leq C, n \to \infty, \Delta t, \Delta X \to 0.$$

Since $\boldsymbol{\rho}(\boldsymbol{E})$ as the spectral radius of $\boldsymbol{E}$ provides a lower bound to any matrix norm, for the scheme to remain stable, we should have $\boldsymbol{\rho}(\boldsymbol{E}) \leq 1$ *o*r equivalently we can say that

$$\left|\frac{1-\theta\Delta t\eta_{\boldsymbol{D}}}{1+(1-\theta)\Delta t\eta_{\boldsymbol{D}}}\right| \leq 1, \tag{21}$$

which holds for $\eta_{\boldsymbol{D}}$, eigenvalues of the matrix $\boldsymbol{D}$, located in the right half plane. Inequality (19) also shows that stability of the scheme (19), in the case of RBFs with shape parameter like MQ, depends upon shape parameter.

5. NUMERIACL EXAMPLES

For two-dimensional option pricing problem under GBM framework, consider the PDE (1) with $d = 2$ and the final time condition for European call-max, $g(S) = \max\{\max\{s_1, s_2\} - E, 0\}$ where E are exercise price in maturity time. The boundary conditions are imposed $V(t, s_1^{max}, s_2) = s_1^{max} - Ee^{-r(T-t)}, V(t, s_1, s_2^{max}) = s_1^{max} - Ee^{-r(T-t)}$, where s_1^{max}, s_2^{max} are respectively maximum of s_1, maximum of s_2. For illustration of the accuracy of the proposed method let $E = 10, r = 0.05, \sigma_1 = 0.22$, $\sigma_2 = 0.14, \rho_{12} = 0.5, T = 0.5$ with 100 time steps, $s_1, s_2 \in [e^{-3.5}, e^4]$ and $\theta = 0.5$. To study the behavior of the original and improved RBF-DQ method, different structured mesh sizes are used with shape parameter $1.25D/\sqrt{N}$ as selected in [15] by Franke, where D is the diameter of the minimal circle enclosing all supporting points. In order to approximate the solution by the original method, we should reduce the BS equation to canonical form in which there is no mixed derivative term (the details can be found in [16].) Figure 1 shows the comparison of numerical results by using original and improved RBF-DQ method in which the root-mean-square-error (RMSE)

$$= \frac{1}{N}\sqrt{\sum_{i=1}^{N}(U_i^{Stz} - U_i^{num})^2},$$

where U_i^{Stz} is the solution by Stulz Method [17] which provides a closed form solution and U_i^{num} is solution by numerical approximations i-th mesh point. Comparison between the CPU-times of two methods has been shown in Figure 2. From the figures, it is clear that the improved RBF-DQ method works much better than the original one. For every simulation, the spectral radius of matrix $\boldsymbol{E}$ is less than unity which guarantees the stability of the scheme.

6.CONCLUSIONS

In this study, the RBF-DQ method in which any spatial derivative is approximated by a linear weighted sum of all the function values in the whole physical domain has been improved to approximate the PDEs with mixed derivative terms. The presented method is used for numerical solution of multi-dimensional option pricing problems in this paper. Numerical results showed that our improved RBF-DQ scheme is an efficient approach for solution of this kind of the PDEs and faster than the original one.

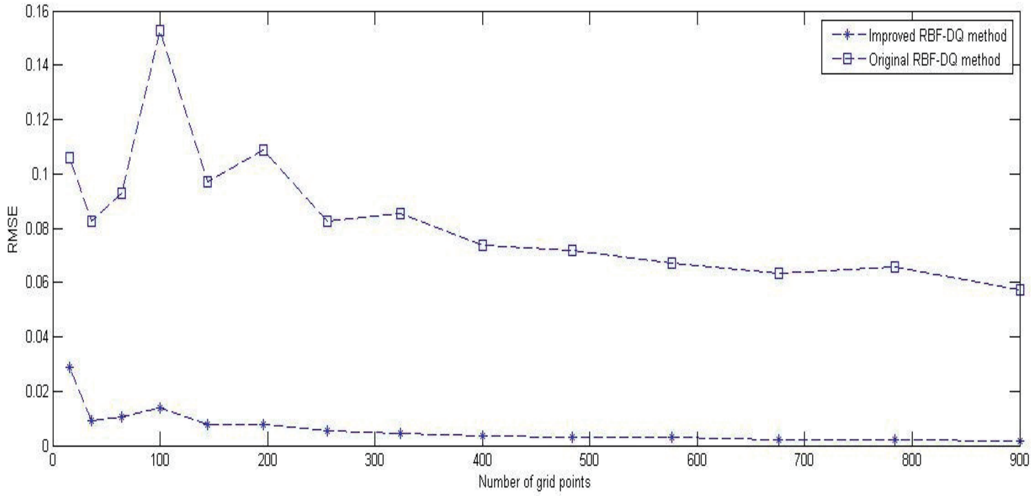

Figure 1. Comparison of root-mean-square-error (RMSE) between the original and improved RBF-DQ method.

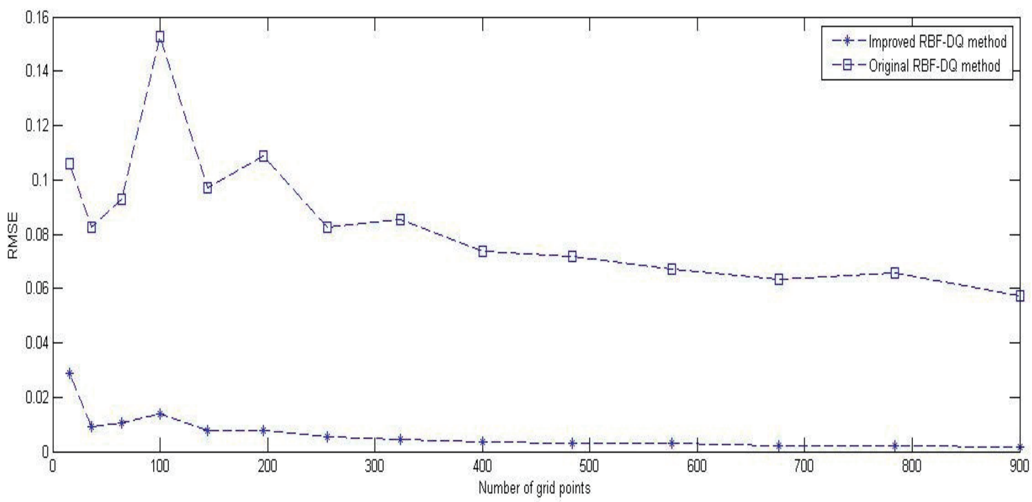

Figure 2. Comparison of CPU-time between the original and improved RBF-DQ method.

References

[1] Bellman R. and Casti J., *J. Math. Anal. Appl.*, 1971; **34**(2): 235-238. DOI 10.1016/0022-247X(71)90110-7.

[2] Bellman R., Kashef B. and Casti J., *J. Comp. Phys.*, 1972; **10**(1): 40-52. DOI 10.1016/0021-9991(72)90089-7.

[3] Shu C., Ding H. and Yeo K., *Eng. Anal. Boun. Elem.*, 2004; **28**(10): 1217-1226. DOI 10.1016/j.enganabound.2003.02.001.

[4] Black F. and Scholes M., *J. Pol. Eco.*, 1973; pp 637-654. DOI 10.1086/260062.

[5] Merton R.C., *Bell J. Econ. Man. Sci.*, 1973; pp 141-183. DOI 10.1142/9789812701022_0008.

[6] Boyle P.P., Evnine J. and Gibbs S., *Revi. Fin. Stud.*, 1989; **2**(2): 241-250 DOI 10.1093/rfs/2.2.241.

[7] Voss D.A., Khaliq A.Q.M., Kazmi S. and He H., *ICCSA*, 2003; pp 199-207. DOI 10.1007/3-540-44842-X_21.

[8] Khaliq A., Voss D. and Kazmi S., *J. Bank. Fin.*, 2006; **30**(2): 489-502. DOI 10.1016/j.jbankfin.2005.04.017.

[9] Fasshauer G.E., Khaliq A.Q.M. and Voss D.A., *J. Chin. Inst. Eng.*, 2004; **27**(4): 563-571. DOI 10.1080/02533839.2004.9670904.

[10] Jo J. and Kim Y., *Bull. Korean Math. Soc.*, 2013; **50**(6): 2035-2051. DOI 10.4134/BKMS.2013.50.6.2035.

[11] Nielsen B.F., Skavhaug O. and Tveito A., *J. Comput. Appl. Math.*, 2008; **222**(1): 3-16. DOI 10.1016/j.cam.2007.10.041.

[12] Martin-Vaquero J., Khaliq A. and Kleefeld B., *Comput. Math. Appl.*, 2014; **67**(6): 1293-1308. DOI 10.1016/j.camwa.2014.01.018.

[13] Micchelli C.A., Interpolation of scattered data: distance matrices and conditionally positive definite functions. Springer (1984). DOI 10.1007/978-94-009-6466-2_7.

[14] Hon Y. and Schaback R., *Appl. Math. Comput.*, 2001; **119**(2): 177-186. DOI 10.1016/S0096-3003(99)00255-6.

[15] Franke R., *Math. Comp.*, 1982; **38**(157): 181-200. DOI 10.1090/S0025-5718-1982-0637296-4.

[16] Jinghui Z., Multi-asset Option Pricing with Levy Process, Ph.D. Thesis, National University of Singapore, 2009.

[17] Stulz R., *J. Fin. Eco.*, 1982; **10**(2): 161-185. DOI 10.1016/0304-405X(82)90011-3.