



# Application of Frequency Domain Experiments in the Factor Screening of Production Systems

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## ABSTRACT

The purpose of this study was to demonstrate that the Application of Frequency Domain Experiments (AFDE) method is effective in screening for discrete variable factors within production systems, plus requires fewer computer simulations than conventional factorial design. To help clarify this, an assembly line production system was designed in order to have factor screening experiments carried out using the AFDE method. The study assembly line produces one type of product and there were ten workstations assigned to be the study's discrete variable factors. Each factor had one to five machines for the workers to control. In the experiment, there were three main study aims in terms of the assembly line production process, these being to carry out: 1) A Major Factor Study, 2) Impact Ranking Major Factors Study, and 3) Sensitivity Impact of the Factors Study.

The study results show that AFDE had the ability to detect bottleneck workstations in the production system for all the types of study undertaken. When assigning the system to have one, two or three bottleneck stations in a respective order, AFDE was able to detect these bottleneck stations and order the importance of the stations with 100% accuracy. In addition, AFDE requires fewer computer runs than conventional factorial design.

**Keywords:** factor screening, frequency domain, simulation.

## 1. INTRODUCTION

Computer simulations are widely used, and have recently become one of the key tools used in operational research. Since a computer simulation allows for the adjustment of factor or parameter values without the need to run any real operation, the simulation outcomes can be used for decision-making prior to running real systems [1]. In addition, Ravindran et al. [2] define 'Computer Simulation' as a numerical

technique used for conducting experiments on computers which involves logical and mathematical relationships that interact to describe the behavior and structure of a complex real world system over extended periods of time.

However, for a very complicated job system such as a job shop, a computer simulation can take a long time to obtain the required results. In a complex job system,

a computer simulation will involve a number of factors, and as a result spend an extended period of time finding a solution. Although there are a number of factors within a complex job system, only a few will influence the response, as stated in Pareto's 80/20 rule. We believe that only 20% to 30% of the input factors influence 70% to 90% of a production system's response; therefore, establishing those influencing factors and reducing the size of the problem should help reduce the time spent analyzing or looking for a suitable solution. Moreover, implementation of the model becomes more convenient, as the complexity of the system is reduced and it is easier to understand. This method is called 'Factor Screening' [3]; for example, if a system involves three factors which are integer numbers from 1 to 10, the solution space in this case will cover  $10^3$  or 1,000 possible answers. However, if there are only two factors, the solution space will cover only  $10^2$  or 100 possible answers.

In general, the conventional factorial design technique is used for factor screening, being considered the most effective method to use, and is conducted by integrating all possible factor levels. Vinod and Sridharan [4] studied factors influencing the scheduling rules in a job shop using the Factorial Design Technique, running a total of 2,160 simulations. In their study there were four studied factors: the average time of an incoming job on two levels, the due date on three levels, time spent installing a machine on three levels, and scheduling rules on twelve levels. In total they conducted ten replications.

The Factorial Design Technique has been extensively used in research, including as a foundation for other types of design which are important in operational process. The most important cases involving factorial design are those involving the number of  $k$ -factors, known as  $2^k$  Factorial Design cases,

in which only two factor levels: 'high' and 'low', are designated. However, using this method to search for major factors within simulations requires many simulation runs; for example, if there are twenty factors one needs to run  $2^{20}$  or 1,048,576 simulations. Thus, in this case if each simulation were to be replicated 30 times, the simulations would have to be run a total of 31,457,280 times. The number of simulations increases according to the increasing exponential distribution, which is an obvious disadvantage of using this method. Some scholars have attempted to reduce the number of simulation runs required; for example Tsao and Liu [5], who studied a suitable number of simulation runs for the  $2^k$  Factorial Design and found that some major factors may be excluded if the number of runs is reduced.

On the other hand, Frequency Domain Methodology (FDM), as introduced by Schruben and Cogliano [6, 7], can screen many factors over only a few computer runs. Jacobson and Schruben [8] also extended the method to include gradient direction estimation, in which a frequency domain method for factor screening is used as the simulation model, and is run with input factors that are varied during a production run according to sinusoidal oscillations. Different frequencies during a run are assigned for each factor, and as the simulation response is sensitive to changes in a particular factor, then the oscillation of that factor induces oscillations in the response. Frequency Domain Experiments (FDE) permit one to identify an appropriate polynomial model for a simulation output. Frequency domain simulation experiments typically require two to three runs to be carried out for factor screening.

Morrice [9] presented a comparison of FDM and the two-level fractional factorial

method, the results showing that where the number of factors is large enough, FDM has the advantage in terms of the power of the test. This methodology has been used extensively in the FDM literature; for example, in Morrice and Schruben [10, 11] and Morrice and Bardhan [12]. In addition, Minsan and Anussornnitisarn [13] stated that FDE can be used for factor screening while the simulated annealing method should be used to search for an optimum. Therefore, using FDE with simulated annealing requires fewer iterations than conventional simulated annealing experiments.

However, FDE can be problematic when using the method with discrete variables such as machines, workers and raw materials, as making adjustments and changes to these variables during the simulation run is quite difficult, that is, there is a need to do system analysis in order to find out when the variables can be adjusted or changed. As a result, this research employed AFDE to screen the major discrete factors within the assembly line production process, as it can be used to screen major factors - just as the  $2^k$  Factorial Design is able to do, but requires fewer runs.

## 2. APPLICATION OF FREQUENCY DOMAIN EXPERIMENTS (AFDE)

Factor screening using AFDE is based on the simulation of a production system. This means that if one wants to study the effects of those input factors that influence the response, one has to create a simulation which represents work behavior in the studied production system. The basic assumptions of the system model are given below.

### 2.1 Basic Assumptions

#### 2.1.1 Polynomial Model

Schruben and Cogliano [7] designated

relations in the system so as to be  $k$ -order polynomial. For this, consider a simulation program with  $p$  input factors  $x_1, x_2, \dots, x_p$ . A response is designated as a variable  $y$ , the expected response value is  $E(Y)$ , as a function of the  $x$ s. Thus, the  $k$ -order polynomial can be represented by Equation 1 below:

$$E(Y) = \beta_0 + \beta_1 \tau_1 + \beta_2 \tau_2 + \dots + \beta_q \tau_q \quad (1)$$

$$\text{or} \quad E(Y) = \beta_0 + \sum_{j=1}^q \beta_j \tau_j .$$

Here,

$E(Y)$  is the expected simulation response;

$\tau_j$  is a term in the  $k$ -order polynomial, that is a particular product of the non-negative integer powers of the input factors, where the sum of the exponents is not greater than  $k$  (e.g.  $k=5$ ,  $x_1^2 x_2^4$  is not such a term because  $2+4$  is greater than 5);

$\beta_j$  is the coefficient for term  $\tau_j$ ; and

$q$  is the number of potential terms in the prospective model.

If the estimated value of a particular  $\beta_j$  is not significantly different from zero, then one can conclude that the corresponding term  $\tau_j$  need not be included in equation (1).

#### 2.1.2 Response Variable

The response variable is collected over a period of time and is represented by the equation below. If  $p$  is the number of factors to be considered over a certain period of time ( $t$ ); for example,  $t$  is a working day, then when  $t$  is increased one working day is added. The response variables in the system for time range  $t$  will not be related to each other and there will be no lag time.

Where  $t$  is the time range 1 to  $n$ , the polynomial terms for all the response functions  $q$  can be represented as follows:

$$\tau_j(t) = \prod_{i=1}^p [x_i(t)]^{e_{ij}}, \text{ where } j=1, \dots, q. \quad (2)$$

Here,

$x_i(t)$  is the input at time  $t$ , where  $i=1, \dots, p$   
and  $t = 1, 2, \dots, n$ ;

$q$  is the total term of all the response functions; and

$e_{ij}$  are positive integers.

The response variable equation will be in the form of a time-invariant linear combination, as follows:

$$y(t) = \sum_{j=1}^q h_j \tau_j(t) + \varepsilon(t), \quad (3)$$

where  $h_j$  is the weighting or impulse response function identical to the input of  $\tau_j(t)$ , and  $\varepsilon(t)$  is the random noise of the system at time  $t$ , where  $t = 1, 2, \dots, n$ .

The equation (3) is called the 'Dynamical Response Surface Model'. There will be no consideration of a time lag in the system; thus, if the response variable can be represented in polynomial equation and the two requirements mentioned above are met, it is possible to conduct AFDE.

## 2.2 Setting the Parameters for AFDE

### 2.2.1 Driving Frequency Assignment ( $\omega$ )

When selecting the driving frequency  $\omega$  (cycles/observation) for the factors in the simulation, every factor must be assigned a different frequency. Where  $n$  is the number of required items of data in an experiment, the analysis of frequency domain spectrum of  $\omega$  will not be precise if its value is lower than  $1/n$ . The maximum value for frequency spectrum analysis is 0.5. The driving frequency for the factors must be in the range  $1/n \leq \omega \leq 0.5$ . In addition, to set a different a value for  $\omega$  for each factor, if one is to consider the interaction terms and  $k$ -order

polynomial terms of the factors; for example the quadratic polynomials, there must be additional methods used.

Jacobson et al. [14] conducted research on the unique frequency assignments for discrete-event simulations - their study investigating 21 factors for quadratic ( $k = 2$ -order) polynomials and eleven factors for cubic ( $k = 3$ -order) polynomials.

### 2.2.2 Selection of Amplitudes for the Driving Frequencies (a)

The selection of amplitudes for the driving frequencies is essential to understand the magnitude of the response spectrum, meaning that the level of response spectrum is proportional to the factor spectrum level of the same frequency. If the amplitude adopted is too small, the resulting oscillation effects will also be too small to be detected. In contrast, if the amplitude is too large, the input factor value will be beyond the possible range. Jacobson [15] researched the influence of amplitude selection for three determinants: 1) feasibility 2) noise, and 3) higher-degree polynomials. The research found that a small amplitude can restrict the factor value within the feasible range when the degree polynomial is low, and that in order to diminish the noise effect, it is important to choose an amplitude that is as large as possible. However, the effect of the three overlapping determinants is not congruent; therefore, when determining the amplitude, consideration of each determinant should be made in this specific order: feasibility, the degree of polynomial and the noise, respectively.

#### I. Determination of a Factor's Range for a Continuous Input Factor.

As with classical experimental designs, the researcher must specify a range of values for each input factor. The experimental

region takes the form of a p-dimensional rectangle:

$$\{(x_1, x_2, \dots, x_p) | L_i \leq x_i \leq U_i\} \tag{4}$$

Here,

$L_i$  is the minimum value of the factor  $i, i = 1, \dots, p$ ; and

$U_i$  is the maximum value of the factor

$$i, i = 1, \dots, p;$$

Amplitudes are selected so that each input is factored over its entire range of values. The value for the input factor  $x_i$ , at simulated time  $t$ , is:

$$x_i(t) = \frac{(U_i + L_i)}{2} + \frac{(U_i - L_i)}{2} \cos(2\pi\omega_i t) \tag{5}$$

### II. Determination of a Factor's Range for a Discrete Input Factor.

However, when considering factors with discrete data, the equation differs from that for (4) and (5), and can be defined as follows:

$$P(x_i(t) = a) = \frac{1}{2} + \frac{\cos(2\pi\omega_i t)}{2} \tag{6}$$

The probability for each factor value,  $x_i(t)$ , is assigned as shown below:

$P(x_i(t))$	Value
0-0.24	$a_1$
0.25-0.49	$a_2$
0.5-0.74	$a_3$
0.75-0.99	$a_4$

where  $a_1, a_2, a_3$  and  $a_4$  are a constant value.

### 2.2.3 Determination of the Length of Experiment (L)

In AFDE, the length of the experiment is independent of the number of factors, which is different from FDF where the length of the experiment is related to the number of factors. The length of AFDE

can be determined according to the simulation model; however, it should be in a steady state after it has passed the warm-up period or transient state. At the beginning of the experiment, the response data can vary and the effectiveness of the system may not be fully revealed. Thus, to make the selection of the factors more reliable, the length of the experiment should be enough to reach a steady state.

### 2.2.4 Determination of the Number of Simulation Runs (n)

The number of simulation runs should be adequate and can be determined according to the run numbers suggested by Jacobson et al. [14], who developed an algorithm to determine the length of an experiment using FDE. However, the suggested method can also be used for AFDE, and yields the same results - as shown in Table 1.

According to Table 1, the driving frequency set is divided by the number of runs; for example, two factors select the driving frequency set {1,4}, so

$$\omega_1 = 1/14 \text{ and } \omega_2 = 4/14, \dots, \text{etc.}$$

According to the set parameter  $\omega, a, L$  and  $n$ , the AFDE can be illustrated in Figure 1.

### 2.3 Computer Simulation

In this experiment, a computer simulation was developed according to the set model, and input factors assigned to be variables according to the parameters for AFDE. Response data was then collected for each simulation run.

### 2.4 Time-Frequency Domain Transformation

From the responses data collected, the time domain data was transformed

to frequency domain data in order to study the spectrum. The Fourier Transform technique was used for the time-frequency transformation, a technique developed by a French mathematician, Jean Baptiste Joseph Fourier (1768-1830) who was born

in Auxerre, France [16].

Fourier analysis is divided into four categories: 1) Continuous-time Fourier Series: CFS, 2) Discrete-time Fourier Series: DFS, 3) Continuous-time Fourier Transform: CFT, and 4) Discrete-time Fourier Transform: DFT.

**Table 1.** Number of runs or replications with the quadratic polynomial equation assigned.

Factors (p)	Set of Driving Frequency	Runs
2	{1,4} {2,3}	14
3	{1,5,8} {1,4,11} {3,4,13} {3,5,12} {8,9,13} {9,11,12}	28
4	{1,4,10,17} {6,8,9,13} {2,3,11,18} {4,5,7,20} {3,5,12,16} {2,9,10,15} {1,6,16,19}	46
5	{1,4,13,19,29} {4,5,7,20,26} {2,5,11,25,26}	69
6	{1,11,28,31,35,49} {3,4,13,28,40,42} {4,9,10,21,37,44} {6,32,40,42,43,47} {8,15,18,20,29,42} {10,12,15,21,28,29} {11,27,35,36,48,50} {16,23,24,43,45,49} {20,24,30,42,45,47}	103
7	{1,41,19,31,44,53,60} {4,7,9,24,30,49,59} {1,9,14,21,40,46,57} {1,7,18,22,27,57,60} {2,3,12,29,37,50,57} {7,9,17,20,32,62,63} {3,7,19,28,30,43,48} {1,10,16,29,34,37,41} {3,8,10,27,41,42,63} {2,17,22,23,30,33,59} {6,9,19,20,36,41,43} {6,31,40,47,51,61,64} {3,21,41,49,50,54,64} {10,11,36,44,49,51,63} {8,33,47,48,51,53,60} {11,16,17,20,46,59,61} {10,11,18,23,37,53,62} {19,29,30,33,54,56,61} {11,14,23,24,29,31,50} {21,27,34,51,56,59,60} {12,17,21,27,40,47,58} {31,32,38,40,43,53,57} {20,33,37,42,43,58,61} {23,28,38,47,49,50,63}	130
8	{10,16,29,33,38,40,41,75}	168
9	{8,30,33,39,40,44,57,59,94}	209
10	{10,12,13,27,31,59,65,94,101,110} {2,15,22,23,47,56,75,83,121,126}	268
11	{9,21,22,26,32,46,55,105,117,135,166}	340
12	{27,44,56,57,59,63,77,102,124,150,155,219}	448
13	{29,37,39,43,44,55,64,77,96,166,208,211,257}	565
14	{29,42,53,57,62,63,65,79,109,140,210,242,269,310}	675
15	{20,28,32,41,42,47,58,65,145,176,179,222,247,298,342}	780
16	{29,38,46,48,49,53,71,83,110,176,217,223,302,358,372,427}	942
17	{37,46,63,66,67,73,78,91,122,160,162,239,309,325,377,430,495}	1,052
18	{37,53,78,86,87,89,93,107,117,155,190,236,255,318,377,453,475,574}	1,208
19	{18,38,42,45,47,53,70,86,96,163,197,218,299,300,372,429,469,589,620}	1,398
20	{15,29,37,40,41,46,64,100,148,167,195,257,291,329,382,447, 535,608,705,707}	1,558
21	{31,32,48,57,59,67,71,100,113,206,221,266,315,389,407,493,570,576,716,767,774}	1,834

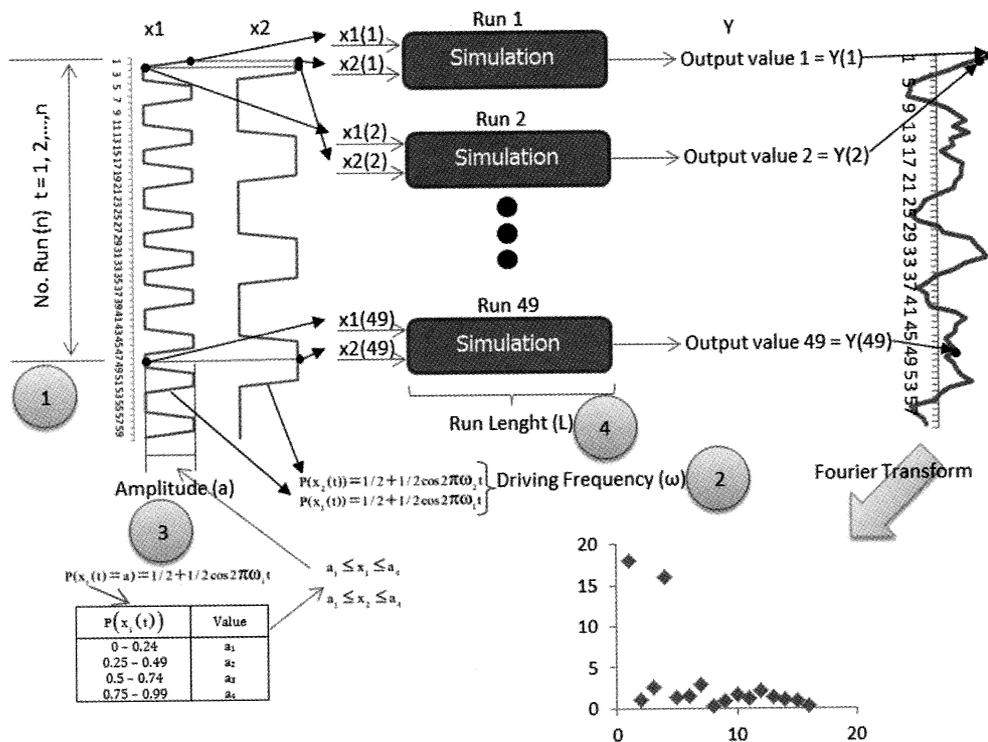


Figure 1. AFDE Process.

In AFDE, the DFT technique is used to transform time to frequency domains, because the received signal is not a period but a discrete amount of time. It is a fact that DFT is effective when is used with the Fast Fourier Transform (FFT) algorithm. In general, the FFT algorithm can make the DFT calculation faster [17].

The transformation of the time domain to a frequency domain can be processed as follows:

$$y(F) = \sqrt{a_{\omega}^2 + b_{\omega}^2}, \text{ where } 1/2 \leq \omega \leq 0.5, \quad (7)$$

$$a_{\omega} = \sum_{t=1}^n (y(t) \cos((t-1)2\pi\omega)),$$

$$b_{\omega} = \sum_{t=1}^n (y(t) \sin((t-1)2\pi\omega)),$$

where,  $y(F)$  is the amplitudinal spectrum of the driving frequency at position  $F$ ,  $y(t)$  is

the response at the simulated time  $t$ ,  $n$  is the number of simulation runs,  $F = \omega n$ , and if  $n$  is an even number then  $F = 1, 2, 3, \dots, 0.5n$  and  $F = \omega(n-1)$ , and if  $n$  is an odd number, then  $F = 1, 2, 3, \dots, 0.5(n-1)$ .

### 2.5 Concluding Method

The analysis and conclusions regarding the significant factors in this study were based on graphs of the spectrum derived from the time-frequency transformation. The technique for graph reading in FDE and AFDE is the same; it is based on the height of the spectrum graph. If one driving frequency of the factor is significant (a high spectrum value), then the input factor is considered as an influence on the response. On the other hand, insignificant driving frequencies are considered conversely.

However, determination of the input factors can also be carried out by using

a statistical analysis with replications, and the response variables to be studied can be tested statistically, as Sanchez et al. [18] have shown in an experiment using four replications and FDE, and comprising a signal run with three replications and a noise run with one replication prior to a statistical test; for example. Nevertheless, the repetition of an experiment will result

in an increased number of experiments; therefore, this research will analyze the influence of input factors through the use of graphs.

**3. EXPERIMENT**

**3.1 Case Study: Assembly System**

This study used an assembly line production process as a model for factor

**Table 2.** Processing time at each workstation.

Scenario	Experiments	Processing Time of Workstation(S) (time unit is seconds)									
		S1	S2	S3	S4	S5	S6	S7	S8	S9	S10
Scenario 1	1	35	10	10	10	10	10	10	10	10	10
	2	10	35	10	10	10	10	10	10	10	10
	3	10	10	35	10	10	10	10	10	10	10
	4	10	10	10	35	10	10	10	10	10	10
	5	10	10	10	10	35	10	10	10	10	10
	6	10	10	10	10	10	35	10	10	10	10
	7	10	10	10	10	10	10	35	10	10	10
	8	10	10	10	10	10	10	10	35	10	10
	9	10	10	10	10	10	10	10	10	35	10
	10	10	10	10	10	10	10	10	10	10	35
Scenario 2	11	35	10	10	10	10	10	10	10	10	22.5
	12	10	22.5	10	10	10	10	10	35	10	10
	13	10	10	22.5	10	10	10	35	10	10	10
	14	10	10	10	10	35	10	10	10	22.5	10
	15	10	10	10	35	10	10	10	10	10	22.5
	16	10	22.5	10	10	10	35	10	10	10	10
	17	10	10	10	10	10	22.5	35	10	10	10
	18	10	10	10	35	10	10	10	22.5	10	10
	19	10	10	10	35	10	10	10	10	22.5	10
	20	10	10	10	10	10	10	10	10	22.5	35
Scenario 3 and 4	21 and 31	35	22.5	16.25	10	10	10	10	10	10	10
	22 and 32	10	10	16.25	35	10	10	10	22.5	10	10
	23 and 33	10	10	10	10	10	10	10	16.25	22.5	35
	24 and 34	10	10	10	22.5	10	10	16.25	10	10	35
	25 and 35	35	10	10	10	22.5	10	10	10	10	16.25
	26 and 36	10	35	10	10	16.25	10	10	10	22.5	10
	27 and 37	10	10	10	35	22.5	10	10	16.25	10	10
	28 and 38	22.5	16.25	10	10	10	10	10	10	10	35
	29 and 39	10	35	10	16.25	10	10	10	22.5	10	10
	30 and 40	10	35	22.5	10	10	10	16.25	10	10	10

screening using AFDE, and the computer software Arena Version 12.0 was used to create the computer simulations for the model. The study assembly line produced one type of product and there were ten workstations assigned to be the study factors. Each workstation had between one and five machines for the workers to control, and the number of machines installed at each workstation was determined by the workers based on their importance. The study aimed to find out which workstations were important, in other words, which were bottleneck workstations in the production line. This meant that if the workstation had only a few machines; for example, one, the workstation would yield a small number of outcomes, and in contrast, if the workstation had more machines; for example, five, more outcomes would be produced out of the workstation. For the unimportant workstations, the number of machines would not affect the production line, or would have little effect on the production outcomes.

### 3.2 Experimental Design

In this experiment, there were three main studies carried out within the assembly line production process: 1) A Major Factor Study, 2) Impact Ranking Major Factors Study, and 3) Sensitivity Impact of Factors Study. The first study was designed for the experiment under scenario 1, the second study was designed for the experiment under scenarios 2 and 3, and the last study was designed for the experiment under scenario 4. For each scenario, the parameters were set for the production system. The set parameters included: 1) Inter-arrival time of materials at the production line's start-point - an exponential distribution with a mean of ten seconds, and 2) the processing time of the machines - a normal distribution with a mean as shown in Table 2 and an SD value

of five seconds for each workstation. The four scenarios used are summarized in Table 2.

#### 3.2.1 A Major Factor Study

This study was used with Scenario 1, in which AFDE was tested for its ability to detect a single significant factor - a bottleneck workstation in an assembly line with ten workstations. Ten experiments (1 to 10) were conducted, and for each experiment a bottleneck workstation was assigned to the experiment - from workstations 1 to 10, respectively.

#### 3.2.2 Impact Ranking Major Factors Study

This study was used with Scenarios 2 and 3, in which AFDE was tested for its ability to detect significant factors, these being the workstations with the longest and second longest processing time. Ten experiments (11 - 20) were conducted, and a random two workstations selected for each experiment to have the longest and second longest processing time. For example, in Experiment 11, workstation 1 was assigned to have the longest processing time (35 seconds), and workstation 10 was assigned to have the second longest processing time (22.5 seconds).

In Scenario 3, AFDE was tested for its ability to detect significant factors, there being the workstations with the longest, second longest and third longest processing times. Ten experiments (21 - 30) were conducted, and three workstations were randomized for each experiment in order to have the longest, second longest and third longest processing times. For example, in Experiment 25, workstation 1 was assigned to have the longest processing time (35 seconds), workstation 5 was assigned to have the second longest processing time

(22.5 seconds), and workstation 10 was assigned to have the third longest processing time.

### 3.2.3 Sensitivity Impact of Factors Study

For Scenario 4, AFDE was tested for its ability to detect significant factors, these being those workstations with the longest, second longest and third longest processing times. At the third order, a break was assigned to the workstation when the process had been running for sixteen hours - the break being eight hours in duration. Ten experiments (31 - 40) were conducted and the parameters of the experiment assigned to be the same as for Scenario 3. For example, in Experiment 31, workstation 3 had the third longest processing time (16.25 seconds). When this workstation had run for sixteen hours, it had to have an eight hour break. There was an attempt made to add complexity to the system by letting AFDE detect a bottleneck workstation which needed to be changed.

The simulation was conducted for 24 hours per day for a total of seven days, and the number of products coming off the production line was studied as a response. The interesting point to note from this experiment is that the workstation had an affect on the studied response, so that the worker involved needed to pay attention.

### 3.3 Arena System

The computer software Arena Version 12.0 was used to create the computer simulations in this model (see Figure 2). The set parameters in the software included: *Inter-arrival Time*

The inter-arrival time of the materials at the production line start-point followed an exponential distribution, with a mean of ten seconds used which, throughout the research, remained unchanged.

#### *Processing time*

The processing time of the machines was assigned to follow a normal distribution, with a mean as shown in Table 2 and with an SD value of five seconds for every workstation. The average processing time per product for each workstation was changed in the program in accordance with the relevant experiment being carried out, and the processing time was divided into four levels, these being:

- 1) Ten seconds - representing normal production levels,
- 2) 35 seconds - representing an additional 25 seconds processing time per product at a workstation, to create the greatest bottleneck conditions for the workstations,
- 3) 22.5 seconds, obtained by reducing the bottleneck time by a half, that is  $10 + (35-10)/2$ , and representing additional processing time at the workstations

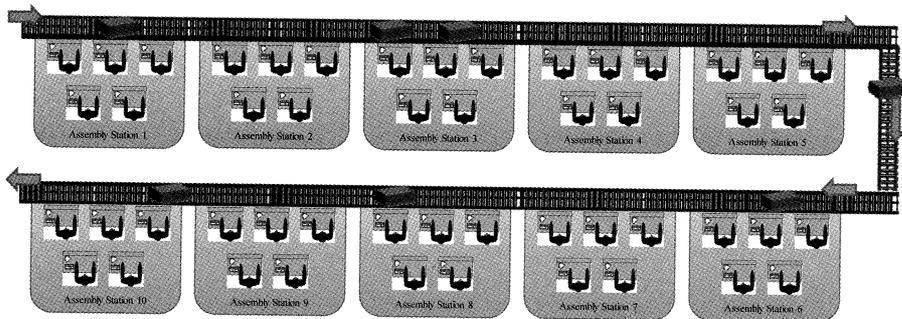


Figure 2. Assembly line production system.

to create a second bottleneck, and 4) 16.25 seconds, obtained based on a reduction in the second bottleneck time by a half, that is  $10 + (22.5-10)/2$  and representing additional processing time at the workstations to create a third bottleneck.

#### *The Number of Machines*

The number of machines at each workstation was assigned as a discrete variable factor to be studied. As a result, a change in the number of machines at each workstation in the Arena program was derived from the driving frequency developed during the AFDE setting process, as discussed in the next section.

### 3.4 AFDE Setting

#### 3.4.1 Driving Frequencies ( $\omega$ )

Non-repeated frequencies were assigned to each of the total ten factors, as follows:

- $\omega_1 = 10/268$  of workstation 1 (S1),
- $\omega_2 = 12/268$  of workstation 2 (S2),
- $\omega_3 = 13/268$  of workstation 3 (S3),
- $\omega_4 = 27/268$  of workstation 4 (S4),
- $\omega_5 = 31/268$  of workstation 5 (S5),
- $\omega_6 = 59/268$  of workstation 6 (S6),
- $\omega_7 = 65/268$  of workstation 7 (S7),
- $\omega_8 = 94/268$  of workstation 8 (S8),
- $\omega_9 = 101/268$  of workstation 9 (S9) and
- $\omega_{10} = 110/268$  of workstation 10 (S10).

#### 3.4.2 Amplitudes (a)

There was only one type of product assembled in this model; the assembly line consisted of ten workstations where each workstation had one to five machines for workers to control. The production control worker was the one who considered the number of machines in each workstation, based on the importance of that workstation. As a result, the minimum amplitude of each workstation or factor was set as 1 and the maximum amplitude of each workstation or factor was set as 5.

#### 3.4.3 Length of each Experiment (L)

This assembly line model ran 24 hours a day for a total seven days; long enough to pass the warm-up period or transient state.

#### 3.4.4 The Number of Simulation Runs (n)

The number of runs was  $n = 268$ . When considering seven days as one run, the total number of runs was 268 weeks, where each week was not related to any other.

With 268 runs, and the product from the production system considered a variable of the studied response, the data for each experiment had to be collected from 268 production systems. The collected data was placed into the time-frequency transformation, and the results from the transformation - 134 frequencies, used to create graphs. After this, the analysis was conducted.

## 4. RESULTS

The study results are divided into four sections, as follows:

### 4.1 Analysis Method

The computer simulation yielded results as shown in Figures 3, 4, 5 and 6. Figure 3 shows the results of Scenario 1, Figure 4 shows the results of Scenario 2, Figure 5 shows the results of Scenario 3 and Figure 6 exhibits the results of Scenario 4.

The data in Figures 3 to 6 is based on the analysis methods shown below.

According to Figure 7, in Experiment 1 - where the y axis is spectral power and the x axis is  $\omega$  (x divided by 268) at different levels, it is apparent that  $\omega_1 = 10/268$  is the most significant. That is, in Experiment 1 workstation 1 was the bottleneck workstation that affected yields. If workstation 1 had fewer machines, the yield would also have been lower. In contrast, if workstation 1 had been given a higher number of machines, the yield would have been higher.

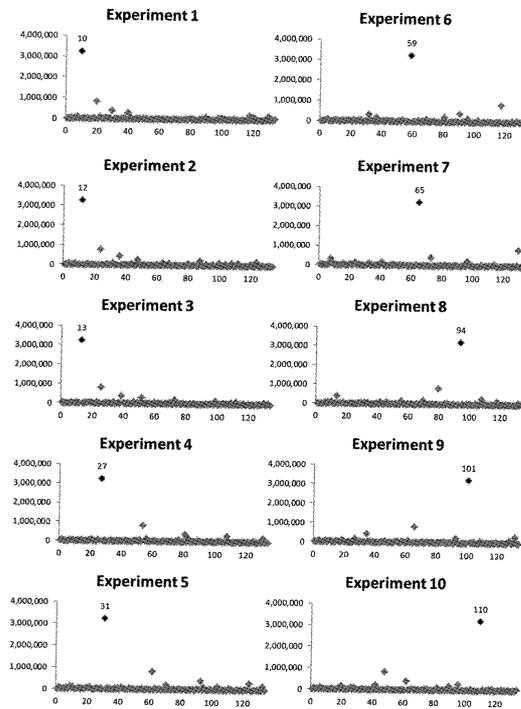


Figure 3. Study results of AFDE for Scenario 1.

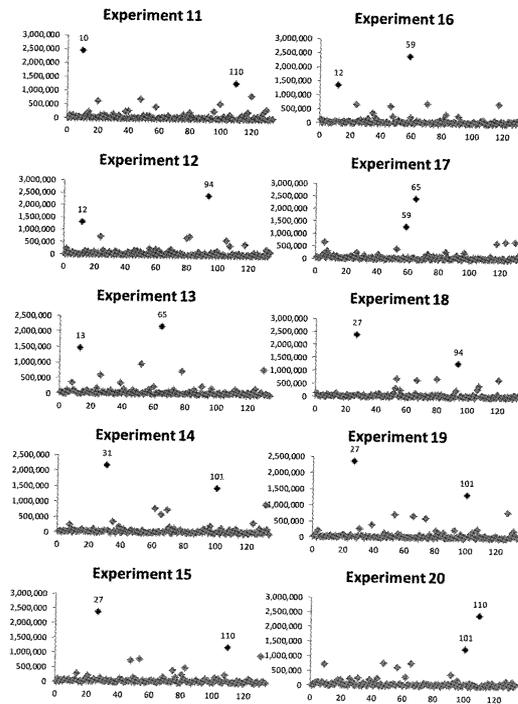


Figure 4. Study results of AFDE for Scenario 2.

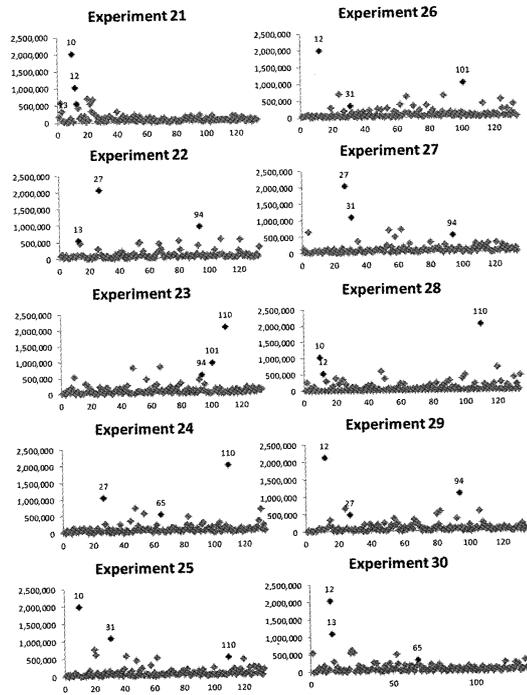


Figure 5. Study results of AFDE for Scenario 3.

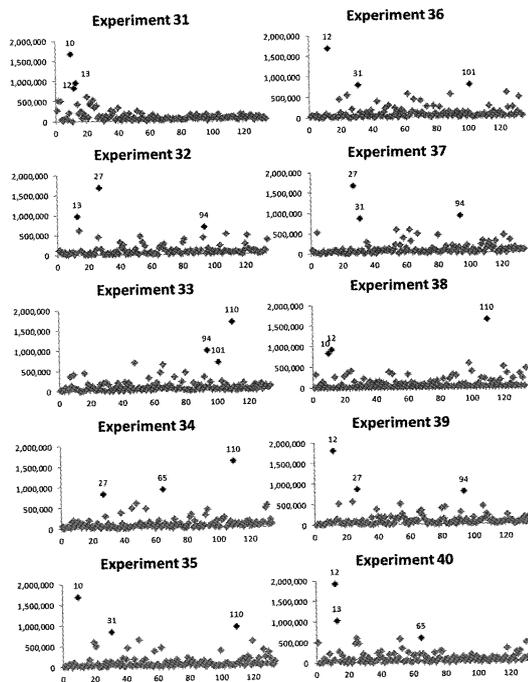


Figure 6. Study results of AFDE for Scenario 4.

According to Figure 8, in Experiment 17 and from the graphs, it is clear that the most significant factor is  $\omega_7 = 65/268$  and the second most significant factor is  $\omega_6 = 59/268$ . That is, in Experiment 17, workstation 7 was the bottleneck workstation that had the most significant influence on the yield, and workstation 6 had the second most significant influence.

According to Figure 9, in Experiment 25 and from the graphs, it is apparent that when considering only the main effect,  $\omega_1 = 10/268$  for workstation 1 was the most significant,  $\omega_5 = 31/268$  for workstation 5 was the second most significant, and  $\omega_{10} = 110/268$  for workstation 10 was the third second most significant. Furthermore, AFDE was able to detect quadratic effects at  $\omega = 20/268, 48/268$  and  $62/268$  for workstations 1, 5 and 10, respectively.

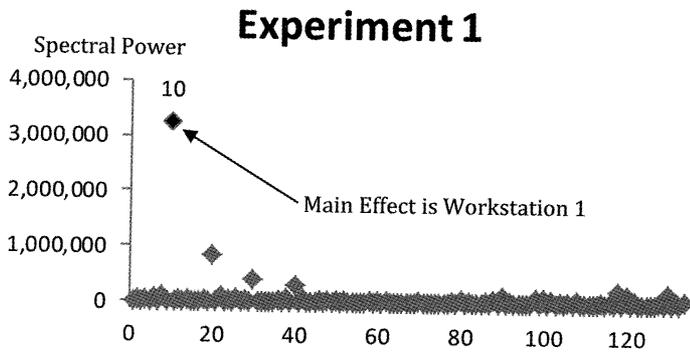


Figure 7. Experiment 1.

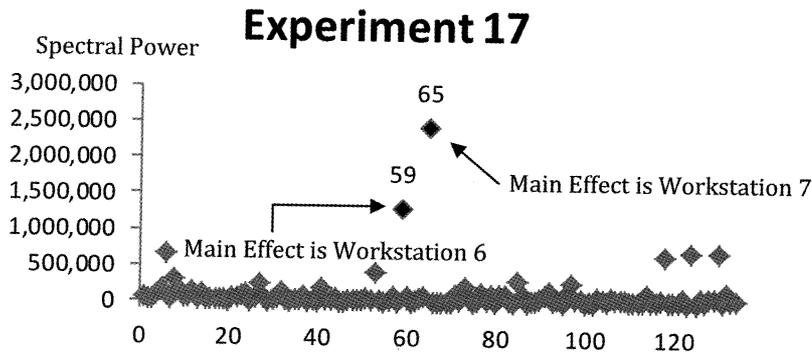


Figure 8. Experiment 17.

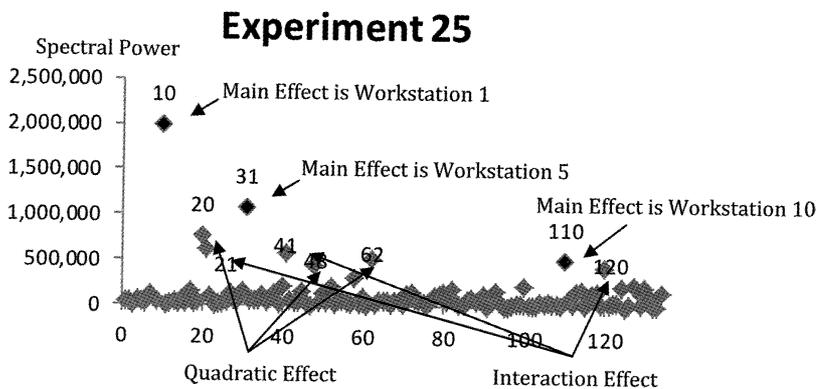


Figure 9. Experiment 25.

It was also able to detect interaction effects among the workstations as follows:  $\omega = 21/268$ , and  $41/268$ , that is the interaction effects of workstations 1 and 5, and  $\omega = 120/268$  the interaction effects of workstations 1 and 10. A study of the main effects, quadratic effects and interaction effects can be used to create a model for forecasting the response variables; however, this study focused most attention on the main effects of the workstations.

The results from Experiments 1, 17 and 25 and other results from the experiment will be discussed in the next topics.

#### 4.2 A Major Factor Analysis

The study results from Scenario 1, as shown in Table 3, show that AFDE detected the ten bottleneck workstations with 100% accuracy. For the workstation with the longest processing time, the driving frequency ( $\omega$ )

of the factor had the most significant spectrum. It is thus apparent that the frequency domain method can accurately screen those discrete major factors that are crucial for the production process.

#### 4.3 Impact Ranking Major Factors Analysis

According to the study results from Scenario 2, as shown in Table 4, when considering Experiment 11 to 20 in Scenario 2, it was found that Experiments 11 to 20 yielded the planned results. For the workstation with the longest processing time, the driving frequency ( $\omega$ ) of the factor had the most significant spectrum. For the workstation with the second longest processing time, the driving frequency ( $\omega$ ) of the factor had the second most significant spectrum. In Scenario 3 of Experiments 21 to 30, as shown in Table 5, the result was also as planned.

**Table 3.** Results of AFDE for Scenario 1.

Experiment	The Longest Processing Time Workstation	AFDE Result for the Bottleneck Workstation	AFDE Corrected
1	S1	S1	100%
2	S2	S2	100%
3	S3	S3	100%
4	S4	S4	100%
5	S5	S5	100%
6	S6	S6	100%
7	S7	S7	100%
8	S8	S8	100%
9	S9	S9	100%
10	S10	S10	100%

**Table 4.** Results of AFDE for Scenario 2.

Experiment	The Longest Processing Time Workstation	AFDE Result for the Bottleneck Workstation	AFDE Corrected
11	S1 > S10	S1 > S10	100%
12	S8 > S2	S8 > S2	100%
13	S7 > S3	S7 > S3	100%
14	S5 > S9	S5 > S9	100%
15	S4 > S10	S4 > S10	100%
16	S6 > S2	S6 > S2	100%
17	S7 > S6	S7 > S6	100%
18	S4 > S8	S4 > S8	100%
19	S4 > S9	S4 > S9	100%
20	S10 > S9	S10 > S9	100%

**Table 5.** Results of AFDE for Scenario 3.

Experiment	The Longest Processing Time Workstation	AFDE Result for the Bottleneck Workstation	AFDE Corrected
21	S1 > S2 > S3	S1 > S2 > S3	100%
22	S4 > S8 > S3	S4 > S8 > S3	100%
23	S10 > S9 > S8	S10 > S9 > S8	100%
24	S10 > S4 > S7	S10 > S4 > S7	100%
25	S1 > S5 > S10	S1 > S5 > S10	100%
26	S2 > S9 > S5	S2 > S9 > S5	100%
27	S4 > S5 > S8	S4 > S5 > S8	100%
28	S10 > S1 > S2	S10 > S1 > S2	100%
29	S2 > S8 > S4	S2 > S8 > S4	100%
30	S2 > S3 > S7	S2 > S3 > S7	100%

**Table 6.** Results of AFDE for Scenario 4.

Experiment	The Longest Processing Time Workstation	AFDE Result for the Bottleneck Workstation	Ranking Change
31	S1 > S2 > S3*	S1 > S3* > S2	Yes
32	S4 > S8 > S3*	S4 > S3* > S8	Yes
33	S10 > S9 > S8*	S10 > S8* > S9	Yes
34	S10 > S4 > S7*	S10 > S7* > S4	Yes
35	S1 > S5 > S10*	S1 > S10* > S5	Yes
36	S2 > S9 > S5*	S2 > S5* > S9	Yes
37	S4 > S5 > S8*	S4 > S8* > S5	Yes
38	S10 > S1 > S2*	S10 > S2* > S1	Yes
39	S2 > S8 > S4*	S2 > S4* > S8	Yes
40	S2 > S3 > S7*	S2 > S3 > S7*	No

\* an 8-hour break after the work was run for 16 hours.

#### 4.4 Sensitivity Impact of Factors Analysis

According to the study results from Scenario 4, as shown in Table 6, the inter-arrival time and processing time parameters were assigned to be the same as in Scenario 3. However, the workstation with the third longest processing time was given an eight hour break after it had run for sixteen hours. The experiment was conducted ten times, that is, Experiments 31 to 40. The results show that the workstation breaks in Experiments 31 to 39 had an effect on production yield, meaning that AFDE can detect the results of workstation breaks. As a result, if a workstation is assigned as having a break, it has more influence on the production system. The bottleneck workstation in the third order became the bottleneck workstation in the second order, and it was only in Experiment 40 under Scenario 4 that the order of the bottleneck workstation was not different from Experiment 30 under Scenario 3. However, in Experiment 40 under Scenario 4, the spectral power of the workstation with the third longest processing time was closer to the workstation with the second longest processing time. It is thus apparent that AFDE is sensitive in terms of detecting the importance of factors.

#### 5. CONCLUSIONS AND RECOMMENDATIONS

In the study, AFDE was able to detect bottleneck workstations in the production system, and when assigning the system to have one, two or three bottleneck workstations in a respective order, AFDE was also able to detect those bottleneck workstations and order their importance - with 100% accuracy. Moreover, AFDE was also able to detect the influence of processing times and machine-breaks on the production system. The workstation where there was a machine break was more crucial in terms of the production

yield. According to this study, it can be concluded that AFDE can detect the impacts on the production system of having both one bottleneck workstation and two or three bottleneck workstations, the interaction between workstations, and the influence that changes in the performance of work stations has on those factors which are the result of any changes in the workstations. For those changes that occurred in the study system, as mentioned previously, although the control worker could not see the changes while he was working, AFDE was still able to detect any significant changes that took place.

It can be concluded that AFDE is an effective factor screening tool, one that has the ability to detect system influences from major factor to single-factor cases, plus the ability to detect and order the major factors in a system that have unequal significance factors. Although there were many interesting factors that could have been studied, only a few simulation runs were conducted, though the results are as good as those when using the  $2^k$  Factorial Design method. According to Table 7, in the study AFDE conducted a total 268 simulation runs for ten discrete variable factors. If this study had used the  $2^k$  Factorial Design method for factor screening,  $2^{10}$  or 1,024 simulation runs would have been required. In the  $2^k$  Factorial Design method, each simulation needs to be replicated, so with two replications there would have been a total of 2,048 simulation runs required. It is apparent; therefore that the use of AFDE becomes more significant when the number of input factors is increased, because the increased number of factors has only a small effect on the number of runs required.

However, it is noticeable that AFDE may not be necessary when screening continuous factors, because the traditional FDE method requires only two to three runs to occur for factor screening. Thus, researchers have to

make a decision on the factor screening method to use according to their simulation requirements. Nevertheless, AFDE can be

used for screening both continuous and discrete factors without needing to study value changes during processing, as with FDE.

**Table 7.** Theoretical comparison of simulation runs.

Factors	No. of Simulation Runs				
	FDM		Conventional Factorial Design		
	FDE*	AFDE	Full Factorial Design (2 Replications)	3 <sup>k</sup> Factorial Design (2 Replications)	2 <sup>k</sup> Factorial Design (2 Replications)
2	≥ 2	14	50	18	8
3	≥ 2	28	250	54	16
4	≥ 2	46	1,250	162	32
5	≥ 2	69	6,250	486	64
6	≥ 2	103	31,250	1,458	128
7	≥ 2	130	156,250	4,374	256
8	≥ 2	168	781,250	13,122	512
9	≥ 2	209	3,906,250	39,366	1,024
10	≥ 2	268	19,531,250	118,098	2,048
11	≥ 2	340	97,656,250	354,294	4,096
12	≥ 2	448	488,281,250	1,062,882	8,192
13	≥ 2	565	2,441,406,250	3,188,646	16,384
14	≥ 2	675	12,207,031,250	9,565,938	32,768
15	≥ 2	780	61,035,156,250	28,697,814	65,536
16	≥ 2	942	305,175,781,250	86,093,442	131,072
17	≥ 2	1,052	1,525,878,906,250	258,280,326	262,144
18	≥ 2	1,208	7,629,394,531,250	774,840,978	524,288
19	≥ 2	1,398	38,146,972,656,250	2,324,522,934	1,048,576
20	≥ 2	1,558	190,734,863,281,250	6,973,568,802	2,097,152
21	≥ 2	1,834	953,674,316,406,250	20,920,706,406	4,194,304

\* FDE can be problematic when using the method with discrete variables such as machines, workers and raw materials, as it is quite difficult to make adjustments and changes to these variables during the simulation run.

AFDE can be used for factor screening in the form of visual experiments and actual experiments, but if AFDE is used in an actual experiment, it might not be convenient in terms of driving frequency assignments, and experimental and calculation methods, and this may confuse the practitioner. AFDE is thus suitable for factor screening in cases where there are many factors and many runs are

required, but it may not be appropriate for an actual experiment. Those who are interested in adopting the method should therefore carefully consider its appropriateness.

AFDE can be used to seek the optimal value in two ways: 1) By searching for the optimal value together with factor screening; for example, when Minsan and Anussornnitisarn [13] applied FDE to screen

continuous factors, they suggested using Simulated Annealing for finding the optimal value, together with reducing the number of continuous factors. This methodology can help reduce the number of searches required if the studied case contains non-significant factors, and

2) By searching for the optimal value after using AFDE for factor screening. Of these two application methods, we suggest that the first is less flexible but requires a low number of searches. The second method is more flexible, because after factor screening has been completed and another method can be used to search for the optimal values; however, this method requires a higher number of searches.

In addition, we recommend that using AFDE is inconvenient at this time, since it requires many calculating programs to be used. If instant software or instant programs are developed, AFDE will become more popular.

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